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THE UNIVERSITY OF ALBERTA

INTERPRETING THE MATHEMATICS CURRICULUM

by

KLAUS PUHLMANN

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF SECONDARY EDUCATION

EDMONTON, ALBERTA

FALL, 1991



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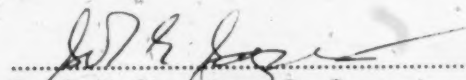
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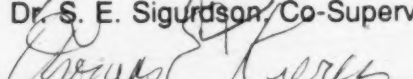
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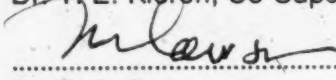
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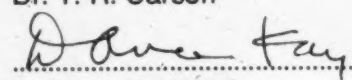
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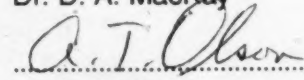
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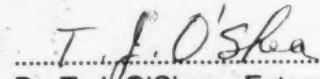

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Abstract

This study examines the interpretive process used by high school mathematics teachers in translating curriculum materials into classroom practice. The emphasis of this study is on **how** teachers interpret the Mathematics 30 curriculum to students and on **why** they interpret the curriculum in particular ways. The data collection and analysis for this study was guided by five major research questions:

1. What interpretation do teachers give of the intended curriculum as represented by the trigonometry unit of the Mathematics 30 curriculum?
2. What are the classroom practices of teachers and to what extent are these practices successful?
3. What are some of the factors which influence the teachers' interpretation of the trigonometry unit to students?
4. What is the relationship between the various factors identified in this study that affect the teachers' interpretation of curriculum materials?
5. What is the nature of teachers' functional paradigm as characterized by their interpretation of the Mathematics 30 curriculum?

The study revealed a number of major findings which are summarized as follows:

1. The data indicate that there is a gap between the intended and teacher-interpreted curriculum. The focus of teachers is on practical matters with an emphasis almost exclusively on content-related goals. This gap is closely related to the Mathematics Diploma Exam (MDE).

2. The findings suggest that mathematics teachers use classroom practices which stress pedagogical efficiency, academic rigor, student understanding and success on the MDE, motivating students, high and clear expectations, feedback on student performance and their own professional competence.
3. The data revealed two major areas of influence on the teachers' interpretation of curriculum materials: training in mathematics and experience in teaching mathematics and the teaching situation which includes the teaching assignment, number of colleagues, school resources, school timetable, nature of students and community expectations. The MDE emerged as a strong and pervasive influence on the teachers' classroom practices.
4. The study identified a number of characteristics that are part of the teachers' functional paradigm. In particular, these include skepticism, assessment-oriented teaching, teacher rhetoric, self-protection, tension, responsibility, maintaining the system and uncertainty/uneasiness.

Acknowledgment

My sincere thanks go to the members of my advisory committee, Dr. Sol Sigurdson, Dr. Tom Kieren, Dr. Al Olson, Dr. Terry Carson and Dr. Al MacKay for their advice and encouragement. In particular, I wish to acknowledge the valuable suggestions, advice and prompt feedback of my supervisors, Dr. Sol Sigurdson and Dr. Tom Kieren. Also the many constructive comments made by Dr. Carson in reviewing the dissertation are very much appreciated. My thanks and appreciation are also extended to Dr. Tom O'Shea from Simon Fraser University for his thoughtful review of the dissertation.

Likewise acknowledged with appreciation are the mathematics teachers who responded to my extensive questionnaire. I am particularly grateful to the three teachers (referred to in the dissertation as Alice, Bill and Carl) who gave their time for the many interviews and the survey and who allowed me to observe and survey their classes. The members of the pilot study review team provided constructive criticism of the survey instrument, observation sheets and interview questions used in the study. A special thanks to John Percevault who assisted me in sorting out the many problems related to data transfer and computer software. My sincere thanks to all of them.

Finally, I wish to thank my wife Sharon, my daughter Tanja and son Ryan. Their encouragement, understanding and loving support made my time away from home easier to bare. I am also indebted to my wife and son for assisting me with the word processing. I dedicate this dissertation to my family.

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I. THE PROBLEM

A. Background To The Problem

Following a period of ten years, during which teachers had complete responsibility for determining the students' final mark and their eligibility for course credits in a particular subject, a new Math 30 curriculum, along with the requirement for students to write a provincially administered diploma examination, was introduced in September 1983.

The new curriculum differed from the previous one not only in its structure, but also in program emphasis. With regard to the structure, the new curriculum has a core and elective component with both components being mandatory requirements.

The difference with regard to program emphasis between the previous and the new curriculum centers around a number of areas. Whereas mechanical solution processes were emphasized in the previous curriculum, the integration of problem solving, applications and statistics using the experimental approach into the overall mathematics program, are emphasized in the new curriculum.

Since the introduction of the new Math 30 curriculum and the diploma examination, teachers have become very conscious of the need to interpret the curriculum in a manner that ensures reasonable success of their students on the diploma examination. While teachers have been regarded as mere "delivery agents" of a new curriculum in the past, there is now considerable evidence that teachers are not neutral in the implementation process. (Connelly & Ben-Peretz, 1980) Implementation of curriculum involves teachers in the adaptation, translation and modification of materials for use in their classrooms. Teachers are also recognizing their critical role in implementing the intended curriculum into classroom practice. This critical role has been acknowledged by a number of curriculum theorists. (Connelly & Ben-Peretz, 1980; Connelly & Elbaz, 1980; Fullan, 1979; Leithwood, 1982) The

implementation process is complex and it is reasonable to assume that there will be discrepancies between the intentions of curriculum developers and the classroom teachers' interpretation of this new curriculum. Invariably, these differences exist because of the different meaning that each attaches to these materials. Consequently, the question of **how** teachers interpret the intended curriculum into classroom practice and **why** they interpret the curriculum in particular ways forms the major focus of this study.

The implementation of the new Math 30 curriculum together with the mandatory provincially administered diploma examination has changed the context in which teachers function. This implementation has the potential of affecting teachers in a significant way. For some teachers the change represents a threat to their prevailing beliefs and for others the proposed change means harmony with theirs. Crocker (1983) points out "... that the context of implementation of innovations may represent a 'strategic site' for the study of functional paradigms. The essential features of such paradigms are much more likely to be laid bare under conditions of change." (p.354)

B. Context of the Study

Alberta Education authorized a new senior high mathematics curriculum in 1983. This new Math 30 course is organized in a "core-elective" format with mandatory components of mathematical subject matter prescribed for all students. The core component is intended to provide a measure of commonality in the instruction and mathematical background of students taking Math 30. The elective component is to allow for the selection of topics to meet student needs or serve as a means of addressing emerging applications. At the completion of this 125 hour course students are required to write a provincially administered Mathematics Diploma Examination. This exam derives its content from the core component of the curriculum only. The elective component is not part of this exam and it is to be evaluated at the school level. The Mathematics Diploma Examination accounts for 50 percent of the students' final mark.

While the previous curriculum enjoyed high levels of teacher satisfaction, curriculum developers in Alberta felt that it no longer reflected the expectations and needs of society. Several changes in content as well as in program emphasis were made. For example, the binomial theorem and mathematical induction were removed as discrete topics and statistics was included. In other instances topics were kept in the curriculum, either receiving greater or decreased emphasis. The unit on trigonometry, as one of six learning units of the Math 30 program, was modified considerably.

The new Math 30 curriculum represents a clear shift in emphasis. While the ability to quantify information and to perform mathematical operations remain important, the new curriculum emphasizes mathematics as a formal study of mathematical concepts and relationships in balance with applied use of mathematics in everyday situations. In this regard, the development and emphasis of problem solving skills is to be prominent in the students' learning experience. The Math 30 program is intended to further develop the problem solving skills developed in earlier grades and practice these in the context of real life situations. The new curriculum challenges and encourages teachers to draw problems from a variety of contexts as concepts are developed in the instructional process. Problem solving was clearly defined, involving a situation in which an individual or group is required to perform a task for which no immediate method or solution is evident. Good mathematical problems should include some of the following characteristics: (1) they involve mathematics in some way, (2) they are of interest to the students, (3) they require the students to interpret and modify the solution process if necessary, (4) they allow for several methods of solution and (5) they allow the student to feel that he or she wants to and can solve the problem. (Alberta Education, Let Problem Solving be the Focus for 1980's, Junior High School) Applications, like problem solving, are intended to be integrated into the overall program rather than dealt with as an independent unit. The use of calculators and computers in the Math 30 program is strongly encouraged in the teaching and learning process.

These are the major changes contained in the new curriculum which

incorporate the recommendations made in An Agenda For Action. (NCTM, 1980) This researcher was extensively involved in the development of this new curriculum by serving on various curriculum and technical committees appointed by Alberta Education. The recommendations contained in An Agenda For Action described a vision of mathematics teaching and learning that was also shared by this researcher. Drawing from his previous teaching experience of Math 30 and other high school mathematics courses, the trend to treat mathematics as nothing more than a repetition of operations involving a series of computations and a number transfer in stated formulas had to be critically examined. (Mathematics 10-20-30 Curriculum Guide, 1983) The committees involved in the curriculum development process drew on the latest research regarding the teaching and learning of mathematics and they incorporated the recommendations made by the National Council of Teachers of Mathematics. When the Math 30 curriculum was finally authorized by Alberta Education for use in Alberta schools, it reflected the mathematics education literature for the 1980's. This implied an emphasis on problem solving, applications, integration of microelectronic technology and flexibility required to respond to the different needs of students.

Following the approval of the Math 30 curriculum, the researcher was extensively involved in developing and conducting inservice activities for teachers. During the inservice sessions and subsequent classroom observations, it was apparent that teachers had wide-ranging interpretations of the new curriculum and its objectives. Having been part of the curriculum development process and knowing what the intent of the curriculum developers (Curriculum Committees) was, teachers clearly deviated in their interpretation from the intent of the curriculum developers.

Through discussions with, classroom observations of and suggestions from teachers, the researcher found that teachers experienced different levels of satisfaction with and acceptance of the new curriculum. Some teachers were unhappy with the increased emphasis on statistics and others regretted the removal of "pet topics", like mathematical induction and the binomial theorem.

The prevailing view of teachers was that the curriculum had been "watered down" in some critical areas, while in others more difficult material was added. Overall, the amount of content was seen as being too much. Some teachers found it difficult initially to "let go" of topics that were no longer in the new curriculum.

The mandatory Mathematics Diploma Examination, which was simultaneously implemented with the new mathematics curriculum, added another dimension to the teachers' working reality. It is reasonable to assume that this exam would affect the teachers' interpretation of the curriculum. That classroom teachers play a critical role in determining curricular outcomes as they translate the intended curriculum into classroom practice has been acknowledged by a number of contemporary curriculum theorists. (Connelly & Ben-Peretz, 1980; Connelly & Elbaz, 1980; Fullan, 1979; Leithwood, 1982; Schwab, 1969, 1971, 1973) Since the intentions of curriculum developers are reflected in the materials that they develop, an examination of the interpretive process used by teachers in deriving their personal meaning of curriculum materials should reveal some of the characteristics of that process. In addition, the findings of such research should contribute toward an improved understanding of the theoretical basis for viewing implementation.

C. Statement of the Problem

With the revised Math 30 curriculum and a mandatory provincial examination in place, the researcher became increasingly interested in the interpretation of the Math 30 curriculum. The study examines the interpretive process used by high school mathematics teachers in translating curriculum materials into classroom practice. While the context of this study is Math 30, the research was confined to the trigonometry unit of that program which served as a particular example of curriculum materials. The emphasis in this study is on **how** teachers interpret the Math 30 curriculum to students and on **why** they interpret the curriculum in particular ways.

D. Theoretical Framework of the Study

The current study focuses primarily on the teachers' interpretation of the intended curriculum with the aim of determining the nature of teachers' functional paradigm as characterized by their interpretation of the Math 30 curriculum. (see FIGURE 1) In addition, it seeks to identify some of the factors which influence teacher interpretation of curriculum materials as well as the interrelationship of these factors. Based on a study by Leithwood & MacDonald, (1981) two major categories of factors are anticipated: those factors related to the teacher and those related to the teacher's current teaching situation. (see FIGURE 2)

This study and the specific research questions are viewed within the context of the following model:

<u>Dimension</u>		<u>Research Focus</u>
1. Curriculum Analysis	Intended Curriculum	Educational System
2. Classroom Process	Teacher-Interpreted Curriculum	School and Classroom
3. Student Outcome	Attained Curriculum	Student

(Taken from Robitaille, O'Shea, Dirks: The Teaching and Learning of Mathematics in British Columbia, Mathematics 8 and Algebra 12, p. 6)

Curriculum Dimension and Research Focus

FIGURE 1

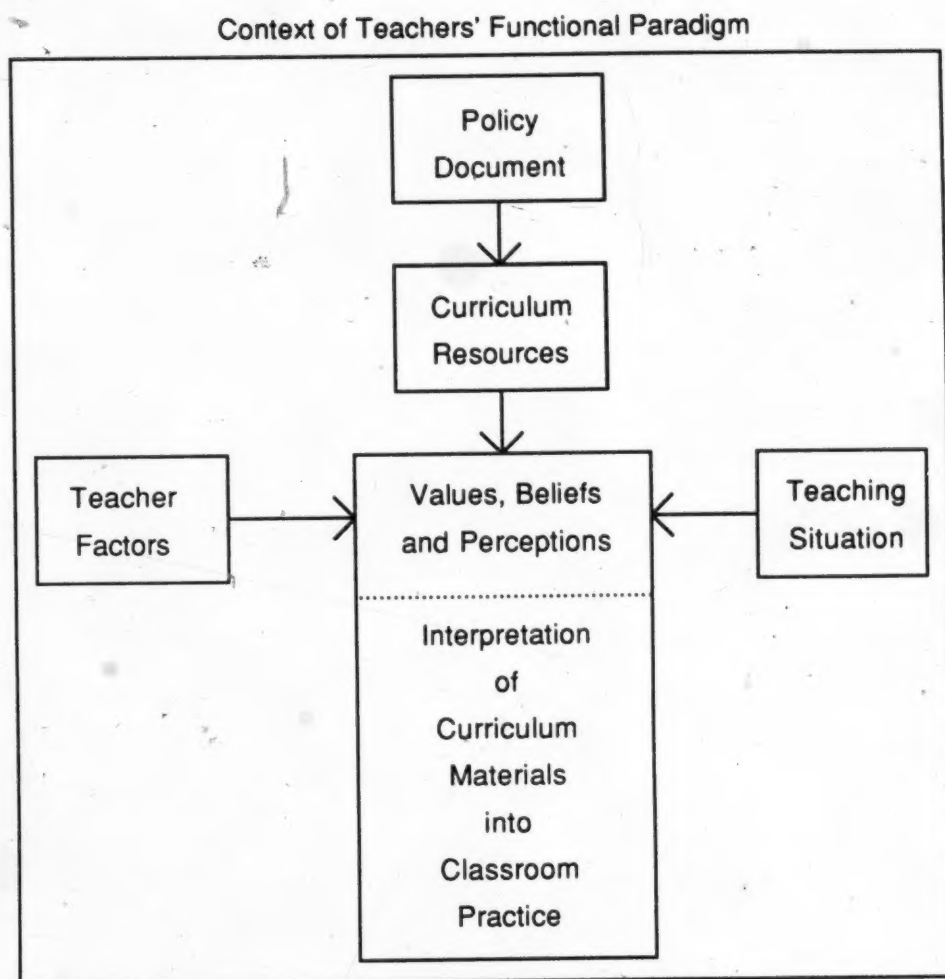
This model views the mathematics curriculum as consisting of three dimensions: (1) the intended curriculum (2) the teacher-interpreted curriculum and (3) the attained curriculum.

The intended curriculum is the Math 30 curriculum as defined by Alberta Education and as outlined in the curriculum guide. (Alberta Education, 1983) This dimension of the model is of some interest in this study. It is examined against the teacher-interpreted curriculum and the attained curriculum.

The teacher-interpreted curriculum is the Math 30 curriculum as taught and spoken of by teachers, taking into account each specific classroom situation. It describes the classroom processes, actions and activities of a teacher which result from giving meaning to the intended curriculum. The teacher-interpreted curriculum is the main focus of this study. Relative to the kinds of interpretations teachers give to the trigonometry unit, the focus is on the nature of the mathematical content, resources selected and used, skills emphasized, teaching strategies and time allocation to various topics/activities. Also included in this dimension is the identification and relationship of factors which influence the teachers' interpretation of curriculum materials or more specifically the reasons for teachers to interpret the curriculum in particular ways. In this study, factors refer to those agents that contribute to an effect. By focusing on the ways teachers interpret the Math 30 curriculum and on the reasons for interpreting the curriculum in particular ways, it allows the factors that influence the teachers' interpretation of the curriculum to be isolated from which inferences about the nature of the teachers' functional paradigm are made. The teachers' functional paradigm is referred to as a constellation of beliefs, values, techniques, exemplars and routines shared by a group of teachers and used by them to guide their classroom practice. (Kuhn, 1970; Crocker, 1983)

The attained curriculum is the Math 30 curriculum assimilated by the students and it is a measure or an indicator of the student outcome.

The teacher factors and the teachers' current teaching situation are two major influences and they are viewed in the context of the following model:



Interpreting Curriculum Material into Classroom Practice

FIGURE 2

This model suggests that the teachers' functional paradigm takes into consideration the curriculum resources, such as books and curriculum guides; the teaching situation, such as students, parents and colleagues; the teacher factors, such as experience and education; and the teachers' values, beliefs and perceptions about school setting, school mathematics, teaching and students.

The study is based on the following three premises:

1. Mathematics teachers' use of curriculum materials is determined by the meaning that these materials have for them.
2. The meaning of curriculum materials for mathematics teachers is shaped through interaction with others. (eg. students, colleagues, supervisors)
3. The meaning of curriculum materials for mathematics teachers is modified through an interpretive process used by teachers in translating curriculum materials into practice in their classroom.

From the teachers' use of curriculum materials one can infer the meaning that these materials have for them and, in turn, from the meanings inferences can be made about the teachers' functional paradigm and how they influence the interpretation of curriculum materials. Meanings, in this study, are taken to be mental creations or images that are formed in and through the defining activities (eg. intended curriculum, curriculum materials) by teachers interacting with these activities. This study examines the meanings that teachers ascribe to the intended curriculum. In this study, the teachers' perceptions of Schwab's (1969, 1971, 1973) four curriculum commonplaces: school setting, school mathematics, teaching and students are used to categorize some of the elements which influence teachers' interpretation of curriculum materials for use in their classroom. A teacher's perception of the physical setting describes that teacher's view of the classroom, school facilities and community position. With respect to the teachers' perception of school mathematics, the focus is on what mathematics ought to be, including content, treatment, expectations and outcome. The teachers' perception of teaching refers to the requirements of effective teaching, including background for teaching and appropriate teaching strategies. Perception of students refers to a teacher's view of the needs, abilities and interests of students.

In an attempt to address the research questions, the researcher adopted two orientations for data gathering: analytical and situational. The analytical

orientation aimed at gathering quantitative data from which certain inferences could be made and through which a better understanding of the teacher's interpretation of the Math 30 curriculum could be reached. The situational orientation of the data gathering was used in case study settings. Here, the researcher attempted to seek the quality of meanings teachers living in a situation give to their lived situation. (Aoki, 1984) The analytical and situational data provided information on how teachers interpret curriculum materials, what factors influence their interpretation, how these factors interrelate and on the nature of the teachers' functional paradigm. The data collected formed the basis for the curriculum material interpretation model.

E. Research Questions

From this broad interest, five specific research questions were derived which define the purpose of this study.

1. What interpretation do teachers give of the intended curriculum as represented by the trigonometry unit of the Mathematics 30 curriculum?
2. What are the classroom practices of teachers and to what extent are these practices successful?
3. What are some of the factors which influence the teachers' interpretation of the trigonometry unit to students?
4. What is the relationship between the various factors identified in this study that affect the teachers' interpretation of curriculum materials?
5. What is the nature of teachers' functional paradigm as characterized by their interpretation of the Mathematics 30 curriculum?

F. Case Study Approach

This study is a multiple-case study involving three individual cases of

Math 30 teachers. In addition, a small sample of six Math 30 teachers was selected from six geographic regions across the province. The purpose of this sample was to situate the three case studies within the context of a small sample of other teachers teaching the same curriculum.

The design of the study incorporated both, qualitative and quantitative approaches in gathering data for the study. Qualitative data were composed of classroom observations and teacher interviews. Each classroom observation was followed by a situational interview of the teacher which was spontaneous in nature and attempted to seek clarification of anything that was observed during the class period. Questions covered such areas as materials and teaching method, reasons for using particular practices, student expectations, nature of the students, student evaluation, student/teacher relationship and perceptions related to mathematics, teaching, students and school setting. Classroom observations, examination of teacher prepared materials, curriculum documents and informal discussions with students provided additional qualitative data.

The quantitative data were derived from administering surveys to the three case study teachers as well as to the sample of six Math 30 teachers from six geographic regions across the province. The survey instrument contained ten parts, each dealing with a particular area of interest. In addition, an attitude test was developed and administered to the students of the three teachers who were observed. Also, the results from the Mathematics Diploma Examination (MDE) for the trigonometry unit for each group of students was available.

The overall purpose of the study was to investigate the interpretive process of Math 30 teachers in translating curriculum materials for use in their classroom. Particular areas of interest in this study were, how teachers interpret curriculum materials, why they interpret curriculum materials in particular ways, what factors influence such interpretation, what is the relationship among these influencing factors and what is the nature of the teachers' functional paradigm as characterized by their interpretation of curriculum materials.

The use of the various instruments and data-gathering approaches served the researcher to gain a fuller understanding of the interpretive process

and to confirm the findings through the use of multiple instruments.

Assumptions

1. It is assumed in this study that the teachers' interpretation of the intended curriculum is reflected in the classroom processes, actions, activities and events, and in the teachers perceptions.

Delimitations

1. The study was delimited to teachers teaching the trigonometry unit as outlined in the Math 30 Curriculum Guide (Alberta Education, 1983). The teachers' interpretation of other units in Math 30 or other subjects was not examined.
2. The study is delimited to an analysis of the data obtained from classroom observations, situational interviews, teacher interviews, surveys of Math 30 teachers, the attitude test and the results from the Mathematics 30 Diploma Examination for the trigonometry unit.

Limitations

1. The study is limited to one unit from the Math 30 program. Generalizations of the findings to other mathematics programs or subjects must be qualified.
2. The study is limited to the information that can be obtained through the instruments, interviews and observations.

G. Need for the Study

The need for this study was derived from a number of realizations. One important realization was the absence of such a study in mathematics classrooms.

With mathematics diploma examinations being a reality and assuming that the diploma examinations are an accurate reflection of the Math 30 curriculum, it is essential that teachers' interpretations of the curriculum, as the connection between the intended curriculum and the actual use of the materials

in the classroom, are somewhat congruent.

An understanding of how the Math 30 curriculum is being interpreted to the students is of considerable value to curriculum developers and practitioners. Such understanding and examination of the interpretive processes used by classroom teachers could lead to a broadening of the theoretical basis surrounding curriculum implementation theory and to the development of useful instruments for such purposes.

H. The Organization of the Dissertation

The dissertation is organized into seven chapters and three appendices. Chapter 1 contains the background to the problem, context of the study, statement of the problem, the theoretical framework of the study, case study approach and the need for the study.

In Chapter 2 a review of related literature to the study is presented. This chapter is organized into four major sections: background to the problem, background to the methodology, research on teacher decision-making and research on mathematics curricula in practice.

Chapter 3 deals with the design of the study. The six sections contained in this chapter deal with the study's questions, the underlying propositions or purposes of the questions, unit of analysis, research instruments and data collection, linking data to the propositions, and criteria for interpreting the findings.

Chapter 4 describes the cases and their context. This chapter includes the Alberta mathematics curriculum, it presents the three case study reports and it reports the findings from the teacher sample.

In Chapter 5 the answers to the first three major research question are presented.

Chapter 6 compares the three cases and situates them within the context of a small sample of mathematics teachers. In addition, the fourth major research question is answered.

Chapter 7 summarizes the research findings with specific emphasis on the nature of the teachers' functional paradigm. This chapter concludes with recommendations for further research and concluding thoughts.

The appendices are structured in the following way: Appendix A contains the Classroom Observation Sheets which also include the Problem Solving Scale. Copies of the Classroom Practices of Trigonometry Teachers Survey (CPTT) and the Attitude Toward Mathematics Test (ATMT) are contained in Appendix B and C, respectively.

II. REVIEW OF RELATED LITERATURE

In order to gain a fuller understanding of how curriculum materials are translated into classroom practice, an examination of the literature related to the topic is necessary. Another purpose of reviewing the literature is to study the background related to the major research questions and to achieve some precision in formulating them. Such a review will also allow sharper and more insightful questions about the research topic to be developed. In addition, it facilitates a better grasp of the research and at the same time allows one to contextualize the findings and understandings derived from the research.

This chapter is grouped into four major categories: background to the problem, background to the methodology, research on teacher decision-making and research on mathematics curricula in practice.

1. Background to the Problem

This section presents and discusses the curriculum standards for school mathematics as outlined by the National Council of Teachers of Mathematics. (1989) In addition, the teaching and learning of mathematics, with an emphasis on the historical development of trigonometry, the acquisition of mathematical knowledge and the teaching-learning process, are presented. Furthermore, the research on teacher's practical knowledge and functional paradigms are discussed.

A. Curriculum Standards for School Mathematics

The National Council of Teachers of Mathematics (1989) published the Curriculum and Evaluation Standards for School Mathematics (referred to as The Standards) which contains a vision for school mathematics for the next decade. This section provides a brief overview of this vision as it relates to the mathematics program, the impact on teaching practices, requirements of

students and school setting factors.

The Standards outlines a vision of mathematics that aims at students becoming mathematically literate both in a world that relies on calculators and computers to carry out mathematical procedures and in a world where mathematics is rapidly growing and is extensively being applied in diverse fields. (p.1) Being mathematically literate implies working toward five general goals for all students: (1) that they learn to value mathematics, (2) that they become confident in their ability to do mathematics, (3) that they become mathematical problem solvers, (4) that they learn to communicate mathematically, and (5) that they learn to reason mathematically. Learning to value mathematics requires that students become aware of the interaction between mathematics and the historical situations from which it has developed. Increasing the students' confidence in their ability requires that school mathematics endows all students with a realization that doing mathematics is a common human activity. Becoming a mathematical problem solver requires students to work on real life applications and problems that take hours and days and that involve small groups or the entire class working cooperatively. It should also involve open-ended problems with no right answer. Learning to communicate mathematically calls for the development of the student's power to use mathematics which involves learning the signs, symbols and terms of mathematics. Doing this in a context of problem situations in which students have an opportunity to read, write and discuss ideas promises the best results. As students communicate their ideas, they learn to clarify, refine and consolidate their thinking. Learning to reason mathematically should provide students with opportunities to make conjectures, gathering evidence and building arguments. This process should receive more emphasis and rewards for students than finding the correct answer.

Becoming mathematically literate or having mathematical power implies a student possessing the ability to explore, to conjecture and to reason logically, as well as to use a variety of mathematical methods effectively to solve problems.

These five goals have a significant impact on the mathematics program, requiring that mathematics becomes more than a collection of concepts and skills to be mastered; it includes methods of investigating and reasoning, means of communication and notions of context. Although it is often necessary to teach specific concepts and procedures, the Standards requires that mathematics must be approached as a whole. Concepts, procedures and intellectual processes are interrelated. It is essential that the mathematics curriculum makes a deliberate attempt, through specific instructional activities, to connect ideas and procedures both among different mathematical topics and with other content areas. The mathematical content calls for a shift in emphasis from a curriculum dominated by memorization of isolated facts and procedures and by proficiency with paper-and-pencil skills to one that emphasizes conceptual understandings, multiple representations and connections, mathematical modeling and mathematical problem solving. Such experiences are essential in order for students to internalize the view that mathematics is a process, a body of knowledge and a human creation.

Inherent in The Standards is the view that all students need to learn more and often different mathematics and that conventional instructional strategies must be significantly revised. Instructional strategies should encourage students to explore, to guess and to even make and correct errors so that they gain confidence in their ability to solve more complex problems. Instructional strategies should require students to read, write and discuss mathematics; to conjecture, test and build arguments about a conjecture's validity. Instruction should evolve from problem situations with an emphasis on "doing" mathematics instead of viewing it as a process in which students passively absorb information, storing it in retrievable fragments as a result of repeated drill and practice. Classroom practices should vary and include opportunities for:

- appropriate project work;
- group and individual assignments;
- discussion between teacher and students and among students;
- practice on mathematical methods;
- exposition by the teacher. (p. 10)

Mathematics instruction must embrace the five broad goals for all students. While The Standards advocates that a common core curriculum be accessible to all students, it does not suggest that the content presentation be the same for all students. It does suggest, however, that the range of content topics be open to all. This requires that teachers select the depth to which a topic is explored according to the level of abstraction at which students are capable of operating. Concrete examples and applications of a topic should be open to all; higher levels of abstraction and generalization should be available to, but not required of, all. The implications are that teachers must provide for differentiation of content which can be achieved through enrichment and extensions of the proposed content rather than by deletion. The Standards suggests that the core curriculum should be extended in a variety of ways to meet the needs, interests and performance levels of individual students or groups of students.

The use of technology has not only made calculations and graphing easier, it has changed the nature of the problems important to mathematics and the methods used to investigate mathematics problems. The impact of technology in instruction is significant because it alters the way teachers teach and students learn. The Standards calls for an increased attention to the use of calculators and computers as tools for learning and doing mathematics. More specifically it is believed that:

- appropriate calculators should be available to all students at all times;
- a computer should be available in every classroom for demonstration purposes;
- every student should have access to a computer for individual and group work;
- students should learn to use the computer as a tool for processing information and performing calculations to investigate and solve problems.

Calculators can and should significantly facilitate the teaching of trigonometry, providing more class time and computational power to develop conceptual understanding and address realistic applications. Computers

provide students with an effective tool to model many realistic problem situations using trigonometric equations or inequalities. They also enhance the students' understanding of the properties of trigonometric functions and their inverses. Of course connected with trigonometric functions are the concepts of amplitude, period and phase shift which can be explored using the concept of transformation. Calculators and computers make mathematics readily available to a wider range of students, thus increasing their mathematical power and literacy.

B. Teaching and Learning of Mathematics

History of Trigonometry

The curriculum guide places considerable emphasis on cultural and historical goals that are to be achieved through the study of mathematics. These goals emphasize the development of the students' appreciation of the contribution of mathematics to the progress of civilization, the development of the students' appreciation of the importance and relevance of mathematics as part of the cultural heritage and the development of the students' appreciation of the role of mathematics in man's total environment.

Trigonometry can be seen from a number of perspectives, each being associated with a specific period in history. If we take trigonometry to mean the analytic science now studied under this name, its origin is properly placed to the seventeenth century. Viewed as a geometric adjunct to astronomy in which certain functions of an angle are used, its real origin lies in the works of Hipparchus (c. 140 B.C.) and even before. The literal meaning of "triangle measurement" dates even further back, the second or third millennium B.C. In his book, History of Mathematics, D.E. Smith (1958) stated that trigonometry experienced a slow growth since the earliest approach to trigonometry appeared in the Rhind Papyrus about 2000 B.C. Reference to the notion of cotangent was made already at that time. Hypsicles of Alexandria (about 180

B.C.) used chord functions of angles in making computations. History seems to credit Hipparchus (about 140 B.C.) with the introduction of trigonometry as a science. In the first century B.C., Heron of Alexandria, a mathematician, introduced the tangent function when making a study of the areas of regular polygons. The contributions of Menelaus of Alexandria (about 100 A.D.) and Ptolemy of Alexandria (about 150 A.D.) were also of considerable significance. While the former dealt extensively with spherical trigonometry, the latter became known for his work in astronomy. The works of both were seen as the authority on the subject for many centuries.

We owe much to the Hindus and Arabs for their contributions to trigonometry. The Hindus replaced the table of chords by a table of half chords (ie. a table of sines). The Arabs, on the other hand, introduced the other functions of angles and prepared tables for them.

The first book on trigonometry alone was *De Triangulis*. It was written by Regiomontanus (Johann Mueller) in 1464 A.D. The book contained accurate tables as well as additions and discoveries of the author. The first book bearing the name "Trigonometry" was written by Pitiscus in 1595. There have been many additions and enrichments to the subject since that time by such mathematicians as Vieta (1540-1603), Thomas Fincke (about 1583), Napier (about 1619), John Newton (1622-1678), John Wallis (1616-1703), Sir Isaac Newton (1642-1727), Jean Bernoulli (1667-1748), Thomas-Fantet de Lagny (about 1710), Euler (1707-1783), Lambert (1728-1777), Kastner (about 1759) and many others.

The growth of trigonometry may have been slow, but today it appears to have a fairly "...permanent position in the world of mathematics because of two seemingly unrelated components or characteristics: (1) its power as a tool for special mensuration, and (2) its power as a precise mathematical model for situations which are periodic." (p.4, Ellis, G. (1976); emphasis in the original)

In terms of its usefulness for measurements, trigonometry plays an important role in astronomy, navigation, surveying, engineering and technology as well as in many trades. The notion of periodicity is dominant in the physical

sciences, biological sciences and economics. The study of periodicity finds its precise mathematical model in trigonometry. The use and application of trigonometry to a wide variety of disciplines is apparent from this brief review of the subject and its historical development. It is also of considerable importance in any mathematics curriculum.

Embedded in the goals of the Alberta mathematics program are several statements that deal with the relevance of mathematics as part of the cultural heritage and its contributions to the progress of civilization. While the statements in the intended curriculum strongly suggest that teachers emphasize this, the teacher-interpreted curriculum as reflected by the activities and practices may well be different. The current study is examining how teachers interpret the intended curriculum to students and why they interpret the curriculum in particular ways.

Visions of Mathematics Teaching and Learning

This section reviews the contributions by some researchers relative to their vision of teaching and learning mathematics. The remarks by Avital and Shettleworth (1968) seem most fitting as an introduction to this section: "Productive thinking in the mathematics classroom, thinking which produces new insights into relationships, begins with the teacher's own awareness of the full range of the mathematical performance." (p. 36)

The questions, "How should mathematics be taught?" or "How can one ensure that the student's mathematical knowledge in a specific topic is properly built?" or "How can mathematical meaning be taught?" are very important in this study. A number of researchers (van Hiele, 1957, 1980; Kieren, 1981; Sigurdson, 1988; Sigurdson and Olson, 1989) have addressed these questions.

In reviewing the van Hiele-based research, Hoffer (1983) states that it "...deals with the interaction between teaching and learning, as well as with an analysis of each." (p.205) The van Hieles developed a model consisting of

three components: insight, thought levels and phases of learning. Finding ways to develop insight in their students was of primary concern to the van Hiele's. A person shows insight if the person (a) is able to perform in a possibly unfamiliar situation; (b) performs competently (correctly and adequately) the acts required by the situation; and (c) performs intentionally (deliberately and consciously) a method that resolves the situation. (p.205) This would imply that to have insight in trigonometry, students understand what they are doing, why they are doing it and when to do it.

The thought levels attached to learning trigonometry or any topic are inductive in nature. The objects studied at level n are extensions of the objects studied at level $n-1$. If students who are thinking at level $n-1$ confront a problem that requires thinking at level n , they are unable to make progress on the problem. These thought levels may also be viewed as categories which contain elements, called objects, and a collection of relations between the objects, called morphisms. It is important that as the curriculum or learning activities are structured according to these thought levels, the categories must not just consist of the objects alone, but it is also necessary to identify the morphisms as well.

With regard to trigonometry, the objects, at each thought level may be organized in the following manner:

LEVEL 0: The objects are symbols that represent, for example, quadrant, origin, axis, coordinate, ordered pair, abscissa, ordinate, unit circle, angles, triangle, hypotenuse, ratio, similarity, congruency, function, domain, range, periodicity, radian, etc.

LEVEL 1: The objects are properties of the symbols, such as conditions for similarity or congruency of triangles or representing trigonometric functions as ratios of two sides in right angle triangles.

LEVEL 2: The objects are statements that relate the properties, such as quotient identities, trigonometric function values for negative angles, law of cofunctions, Pythagorean identities, reciprocal identities or trigonometric equations.

LEVEL 3: The objects are sequences of statements, such as proving the

various identities or deriving the law of sine and cosine.

LEVEL 4: The objects are properties that analyze the logic of different systems, for example, by comparing plane trigonometry with spherical trigonometry.

Phases of learning constitute the third component of the van Hiele model. These phases are necessary to move students from one thought level to the next within a subject. A sequence of five phases of learning have been identified which serve as a prescription for organizing instruction. These phases are: inquiry, directed orientation, expliciting, free orientation and integration. The phases of learning entail the following activities:

Phase 1: Inquiry - The teacher engages the students in (two-way) conversations about the objects of study. The teacher learns how the students interpret the words and gives the students some understanding of the topic to be studied. Questions are raised and observations made that use the vocabulary and objects of the topic and set the stage for further study.

Phase 2: Directed Orientation - The teacher carefully sequences activities for student exploration by which students begin to realize what direction the study is taking, and they become familiar with the characteristic structures. Many of the activities in this phase are one-step tasks that elicit specific responses.

Phase 3: Expliciting - The students, building from previous experiences, with minimal prompting by the teacher, refine their use of the vocabulary and express their opinions about the inherent structures of the study. During this phase, the students begin to form the system of relations of the study.

Phase 4: Free orientation - The students now encounter multistep tasks, or tasks that can be completed in different ways. They gain experience in finding their own way or resolving the tasks. By orienting themselves in the field of investigation, many of the relations between the objects of the study become explicit to the students.

Phase 5: Integration - The students now review the methods at their disposal and form an overview. The objects and relations are unified and

internalized into a new domain of thought. The teacher aids this process by providing global surveys of what the students already know, being careful not to present new or discordant ideas. (p. 208) At the close of the fifth phase, the new level of thought is attained and students have developed insight in the subject matter studied.

Kieren (1981) is another researcher who has contributed significantly to the question of how to teach mathematics that ensures mathematical knowledge building. Based on his extensive research on rational numbers, he developed a theory that states that the foundation of knowing any mathematics or of producing a well-developed mathematical knowledge system depends on having five kinds of necessary experiences. Referring to these as the five faces of mathematical knowledge building, he describes the nature of these five faces as follows:

1. The first face exposes the mathematical nature of the knowledge. It shows the important mathematical variations or representations of an idea.
2. The second face is developmental in nature. It involves the schema, mental structures and information processing capacities necessary to work with various levels of mathematical ideas.
3. The third face is constructive in nature. It shows the mechanisms useful in extending the images and building up a mathematical idea.
4. The fourth face is visual in nature. It involves experiences with the images or mental pictures of an idea. Mental images likely must transcend the "visual", but the concept of "image" is still useful.
5. The fifth face is symbolic in nature. It brings to view the various formal and informal language structures, oral and written, words and mathematical symbols which a person uses to describe and work with the idea or knowledge. (p. 4-5, emphasis in the original)

Kieren maintains that mathematical knowledge building involves the coordination of these five faces and teachers must provide opportunities and experiences which allow the faces to develop. In terms of curriculums, texts, and instructional materials, it is implied that instruction must deliberately provide

for the development of the first, third, fourth and fifth face for each learner and must take into account a learner's status with regard to the developmental face two. (p. 5)

When comparing the five faces of mathematical knowledge building with the van Hiele model, one can see remarkable similarities. The "nature of mathematics" face shows important mathematical variations or representations of an idea. Students have opportunities to disclose their intuitive knowledge about a concept and at the same time they are exposed to the various connotations of that concept. Students deal with the concept in global terms without specifically identifying its properties. Kieren and van Hiele seem to suggest that the mathematical experiences of students are totally qualitative in nature at this level.

The "constructive" face seems to correspond to level 2 of the van Hiele model. Students begin to identify various properties embedded in a concept and they begin to relate these properties. This face or level is important in extending the student's images and building up a mathematical idea. Kieren contends that in building mathematics curriculum for students, we have generally not clearly uncovered useful constructive mechanisms and definitely not included them in the curriculum.

The "visual" face is seen as important in that mental images help the student define the idea for himself or herself. Such images, however, should broaden the student's view rather than being narrow or limiting. As an example, in trigonometry it is important that students have a variety of graphing experiences (eg. developing images of how $\sin(nx)$ changes as the value for n changes or experiences with periodicity) or are exposed to the different interpretations of trigonometric functions (eg. Ratio Interpretation, Unit Circle Interpretation, Wrapping Function Interpretation, Line Representation of Trigonometric Functions, Infinite Sequence Interpretation, Vector Interpretation or Complex Number Theory Interpretation). It is clear that each interpretation would generate different images, thereby enriching the student's mathematical experiences.

The "symbolic" face is important from the point of view that students must become acquainted with the standard symbols used in mathematics. Kieren maintains, however, that "as precise standard symbols are introduced, it must be noted by the teacher and experienced by the learner that this new symbolism has many connotations and can be associated with each of the informal languages learned and used." (p. 8)

The focus of Sigurdson and Olson (1989) was more on the question of "How can mathematical meaning be taught?" and "How can contextual learning contribute to meaning construction in mathematics?". (Sigurdson, 1988) The role and importance of "meaning" in the process of teaching mathematics is a dominant preoccupation among practitioners. In order to understand the impact of meaning-oriented instruction, Sigurdson and Olson devised a research project that involved four instructional models being used to 54 grade 8 classes. The instructional models were Conventional Textbook Instruction (CTI), Direct Instruction (algorithmic-practice) (DI), Meaning Instruction (MI) and Problem-Process Instruction (meaning plus a problem process focus) (PPI). The major objective of this study was to compare these instructional approaches, using as a criterion for this comparison, mathematical achievement and attitude to a mathematics class of grade 8 students. The study had two other objectives relevant to the current study. These were:

1. Are teachers in the context of offering a provincial curriculum able to implement each of the three innovative treatments appropriately?
2. What are some curriculum, school, teacher, class and student factors that influence the implementation of the three instructional models?

With regard to the effectiveness of the instructional approaches relative to mathematical achievement and attitude, the conclusions seem to support that meaning instruction is very effective in teaching mathematics, particularly for a well-implemented approach. Where a lower level of implementation of this approach was evident, only average and above-average students benefit. Sigurdson and Olson could only speculate as to why this was happening.

Furthermore, the emphasis on a ten-minute daily focus on problem-process-focused instruction was not supported by their findings.

Relative to the teachers' ability to implement each of the three innovative treatments within the context of the provincial curriculum, the researchers found great differences. Teachers found that the difficulty level increased from Direct Instruction, Meaning Instruction, to Problem Process Instruction. This observation is relevant to the current study in that one of the research questions seeks to determine the extent to which teachers' practices are successful in the classroom.

The identification of factors that influence the implementation of the three instructional approaches is also of major interest to the current study. In their study, Sigurdson and Olson found major differences in the level of implementation among teachers, however, in terms of identifying factors that influence the implementation, they were only able to point at the general ability of the class as a factor affecting the implementation/achievement.

Further to the question of, "How can mathematical meaning be taught?", Sigurdson (1988) argues that children learn and retain arithmetic processes better when they are taught meaningfully. He added that even researchers at the forefront of analyzing teachers' classroom behavior are convinced that 50% of lesson time should be spent on "introducing and explaining mathematics concepts", "a process whereby teachers facilitate the meaningful acquisition of an idea."

In reviewing the various visions of mathematics learning, Sigurdson stated that the current interest in "teaching thinking skills, learning skills, or higher order cognitive abilities" has led to the issue of learning in context. The skills, knowledge and mathematical procedures take on meaning in the context of the whole. Successful mathematics learners engage in more metacognitive behaviors, such as checking their own understanding of procedures, monitoring for consistency and trying to relate new material to prior knowledge. Most teachers simply let context happen. They use it when they or their students are in trouble. They use it to bring emphasis. They use it for providing motivation or

for stimulating involvement. Sigurdson concludes that context should be used more aggressively in teaching mathematics, because anything that shows so prominently like context in many different visions of mathematics learning must point to a truth about instruction.

C. Teachers' Practical Knowledge

The teachers' interpretation of curriculum materials to students is often studied by focusing on the teacher's actions, activities and decision-making during the actual teaching and learning process. Such studies make invariably three assumptions relative to the meaning of curriculum materials to teachers. It is assumed that a mathematics teacher's use of curriculum materials will be determined by the meaning that the materials have for the teacher, not by curriculum material per se. Furthermore, it is assumed that the meaning of the curriculum materials will be shaped by the interactions of the teacher with others. Thirdly, the meaning of the curriculum materials used by the mathematics teachers is modified through an interpretive process used in translating curriculum materials into practice. Implicit in these assumptions is that the teachers' theoretical knowledge is different from their practical knowledge.

Connelly and Ben-Peretz (1982) have examined the role of the classroom teacher in using and doing curriculum research and curriculum development over the past twenty years. They concluded that "...teachers are not properly seen as implementers of research and development ideas nor as assistants to research in any sense, but are, rather best seen as independent agents with an educative function in curriculum development and implementation." (p. 209) Their view about the future is somewhat more optimistic in that they see a growing trend to move away from the image of teachers as neutral transmitters of curricular programs and toward a more sophisticated notion of the teacher as active implementer.

In the past, teachers may have been bypassed in the planning,

development and implementation of curriculum, however, it remains to be seen whether they remained neutral agents in the classroom. Connelly and Elbaz (1980) maintain "...that teachers do not neutrally implement theories and programs; they develop programs of study for their classrooms by adaptation, translation, and modification of given programs and research findings; they may even occasionally develop their own curriculum materials." (p. 107)

In the same study, Connelly and Elbaz, strongly advocate shifting the focus of research to the teacher and the act of teaching. They present a strong case for studying teachers' practical knowledge. The teacher's practical knowledge is not seen by them as simply a collection of random theoretical statements combined with handy hints for feeding students. Rather, they suggest that the teacher's knowledge can most usefully be viewed as a body of knowledge, not a totally comprehensive or consistent one to be sure, but nonetheless a body of knowledge held in a uniquely practical way and structured in terms of the teacher's practical purposes. (p. 113)

Based on a case study, Connelly and Elbaz (1982) present five orientations which seem to structure the practical knowledge of one particular teacher: to situations, to theory, to others (social), to self (personal) and to experience. Each of these orientations seem to have three levels of organizations: (1) rules of practice, (2) practical principles and (3) images. Connelly and Elbaz drew the following conclusions about these three levels of organizations. Rules of practice are statements, sometimes highly descriptive, of what the teacher does. The use of rules of practice is a methodical carrying out of the teacher's purposes, which may or may not be articulated. The practical principle is a broader, more inclusive, statement than the rule. Practical principles embody purpose in a deliberate and reflective way; the statement of a principle enunciates, or at least implies, the rationale which emerges at the end of a process of deliberation on a problem. Finally, the image is a brief, descriptive, and sometimes metaphoric statement which captures some essential feature of the teacher's perception of himself, his teaching, his situation in the classroom, or his subject matter. Images serve to

guide the teacher's thinking and to organize knowledge in the relevant area. The image is generally imbued with a judgement of value and constitutes a guide to the intuitive realization of the teacher's purposes. (p. 117-118)

While Connelly and Elbaz have argued that the teacher's curriculum thought is practical and that it arises from experience in a unique, personal - though not unprincipled - way, the existence of common elements in teachers' practical knowledge is not ruled out. Teachers are known to identify with experiences of other teachers.

In his book, The Reflective Practitioner, Schön (1983) deals extensively with the question of how professionals think in action. He maintains that our knowing is ordinarily tacit, implicit in our patterns of action and in our feel for the stuff with which we are dealing. Our knowing is in our action. In the day-to-day practice of the professional, it is quite common for the professional not to be able to state adequate criteria for his judgements or display skills for which he cannot state the rules and procedures. This implies that the professional depends on his tacit knowing-in-action. Schön is not suggesting that intelligent action can not be accompanied by some kind of knowing. The tacit knowing-in-action refers to examples like recognizing the mood of a person without being able to tell or knowing a person's face and then recognizing it among a thousand faces.

Likewise with reflection-in-action, Schön recognizes that we sometimes think about what we are doing while we are doing it. It is like "thinking on one's feet." However, in examining the patterns and limits of reflection-in-action, he concluded that even if reflection-in-action is feasible, it tends to interfere with action. It may seem to do so for four different reasons:

1. There is not time to reflect when we are on the firing line; if we stop to think, we may be dead.
2. When we think about what we are doing, we surface complexity, which interferes with the smooth flow of action. The complexity that we can manage unconscious paralyzes us when we bring it to consciousness.

3. If we begin to reflect-in-action, we may trigger an infinite regress of reflection on action, then on our reflection on action, and so on ad infinitum.
4. The stance appropriate to reflection is incompatible with the stance appropriate to action. (ie. reflection that is not knowledge producing interrupts) (p. 277- 278)

Based on these points, it appears that reflection-in-action is a contradiction in terms. Yet Schön insists that practitioners do reflect-in-action when they are stuck or seriously dissatisfied with their performance. In this discussion the possibility arises of there being a considerable difference between the teachers' action in use and their espoused theories about their action.

D. Functional Paradigm

In his book, The Structure of Scientific Revolution, Kuhn (1970) uses the term "paradigm" in two different senses.

On the one hand, it stands for the entire constellation of beliefs, values, techniques, and so on shared by the members of a given community. On the other, it denotes one sort of element in that constellation, the concrete puzzle-solutions which, employed as models or examples, can replace explicit rules as a basis for the solution of the remaining puzzles of normal science. (p. 175)

Extending Kuhn's (1970) notion of paradigm and paradigm shifts, Imershein (1977) argued that organizational change may be considered analogous to scientific change and that the activities of organizational members may be seen as paradigm guided in a manner similar to scientific activities. He bases his reformulation of the paradigm concept on three major points in Kuhn's analysis of the development of science.

1. In learning scientific practices, the student does not learn a set of rules or abstract theories which he then applies in research, rather he is introduced to a common set of practices, instrumentations, research questions, and the like, all of which are provided by the current paradigm. Scientific practices are learned by relying upon and employing concrete examples of successful scientific research which serve as the exemplars of a particular paradigm.
2. Scientific change advances by revolutions from one paradigm to another. It changes ways of thinking, seeing the world and doing research. This paradigm shift occurs at a point when a paradigm is perceived to be more effective in handling old and new as well as unresolved research problems. The perception of these unresolved anomalies typically sets the stage for paradigm revolution.
3. The specification and organization of scientific activities is paradigm specific. That is, the nature of the research and how it is to be carried out are part and parcel of the paradigm itself.

Imershein further suggests that the research emphasis should be placed on directing greater attention toward the content and structure of knowledge used by organizational participants as well as in providing an alternative model of understanding organizational action.

In his article, *The Functional Paradigm of Teachers*, Crocker (1983) makes some useful contributions with regard to the study of functional paradigm of teachers. He states:

Following Imershein's line of argument, it is assumed that teachers are, indeed, similar to other communities of scholars or practitioners in that they share common goals, problems, exemplars, routines, etc. which constitute a "functional paradigm." The use of the term "functional paradigm" is meant to convey the idea that the characteristics which unite a community of practitioners are likely to be centered on practical matters. (p. 354)

Crocker advocates that the implementation of innovations represent a "strategic site" for the study of teachers' functional paradigm.

While Crocker clearly suggests that identifying the functional paradigm of teachers involves focusing on commonalities among teachers rather than

differences, the notion of common elements in teachers' practical knowledge is not universally accepted. Lortie (1975) concludes that teachers do not see themselves as sharing "a viable, generalized body of knowledge and practice." (p.79) He further states that teaching is "marked by the absence of concrete models for emulation, unclear lines of influence, multiple and controversial criteria, ambiguity about assessment timing and instability in the end product." (p.136) Also Connelly and Elbaz (1980), in their one teacher case study, summed up their feeling by saying that the structure of a teacher's practical knowledge is likely to be unique. It seems that both are focusing on the first part of Kuhn's definition which defines paradigm as the entire constellation of beliefs, values, techniques, and so on shared by the members of a given community. However, when the focus shifts to the second part of Kuhn's definition, reference to "shared examples" or a "decision-making system" used by all teachers is appropriate.

2. Background to the Methodology

In this section the literature on research methodologies is being reviewed. The focus will be on reviewing three research orientations, the case study as a research strategy, the concepts of validity and reliability as they relate to qualitative and quantitative research and research methodologies used in studies of classrooms.

A. Research Orientations

The literature review seems to indicate that the debate concerning quantitative versus qualitative research approaches is continuing. However, Aoki (1984) points out that in recent years we have come to question the tendency of many educators to reduce the understanding of educational evaluation to the paradigm of scientific research. Drawing on the ideas of Habermas (1972), Aoki (1984) outlined three orientations in curriculum

research. The first is the ends-means orientation which reflects an achievement, goal based, criterion referenced and cost benefit oriented mode of evaluation. The researcher comes to know through empirical data. Second, there is the situational interpretive orientation in which the researcher seeks the quality of meanings people living in a situation give to their lived situations. In this orientation, coming to know or reaching an understanding is in terms of the structure of meaning. The third orientation is the critical theoretic orientation. It involves discovering through critical reflection, the underlying human conditions, assumptions and intentions and improving or transforming same. The form of knowing is critical knowing in the sense of understanding hidden assumptions, perspectives, motives, rationalizations and ideologies.

McCutcheon (1981) examined the nature of the interpretive endeavor. She views interpretation as a transaction between the researcher's knowledge and the observations being made, and therefore as placing a researcher in an active role in the construction of meaning. In referring to the three approaches to curriculum inquiry, she suggests that they can be seen as forming a triangle. One can place a particular research study along any side or even within the triangle, by the degree to which it reveals the assumptions and principles underlying each of the three approaches or blends them together. Distinctions can be made among them by the methods each employs, the questions to be addressed, and the assumptions regarding issues such as generalizability and objectivity/subjectivity. A researcher using a qualitative approach based on assumptions and principles of phenomenology would be likely to interpret events in light of the meanings participants make of those events, whereas a researcher using a qualitative approach based on assumptions and principles of critical science might interpret events in light of wider theoretical considerations, then critique the phenomenon in light of those considerations, calling for changes in practice or theory.

McCutcheon illustrates the three research orientations by considering how researchers of each orientation might approach a study of 42-minute class periods.

A positivist studying this situation might wonder how to increase teacher's effectiveness, and thereby student achievement, in the 42-minute periods, perhaps by measuring students time on task and correlating that with achievement test scores. A phenomenologist might wonder what meaning 42-minute periods have for participants. A critical scientist might wonder about the origin of 42-minute periods and how such time affects student's experience, rendering disciplines accessible to some and inaccessible to other students, thereby reproducing society so that the able remain able and the weak remain weak. (p.6)

According to McCutcheon, the research methodology appropriate for a study will depend upon the questions asked by the researcher.

Smith and Heshusius (1986) indicate that there is a trend in the literature concerning the debate on quantitative versus qualitative approaches to research which suggests two things. First, many educational inquirers now accept the idea that there are two different, equally legitimate, approaches to inquiry. Second, many inquirers also feel that whatever differences may exist between the two perspectives, they do not, in the final analysis really matter very much. What they are saying is that many educational inquirers now seem to think that the profession has reached a stage of, if not synthesis, then certainly compatibility and cooperation between the two approaches. This view is certainly expressed by Cronbach (1980) who is being quoted by Smith and Heshusius:

The demand that an inquirer be "either/or" has been replaced by the injunction to employ both approaches in combination or to draw on both styles at appropriate times and in appropriate amounts. (Smith and Heshusius, 1986, p. 223)

Smith and Heshusius argue that at the level of applying specific individual procedures, quantitative inquirers can supplement their controlled instrumentation with open-ended observation in naturalistic settings and qualitative inquirers can supplement naturalistic observation with the quantification of events. In both cases, but especially for qualitative inquiry, the logic of justification does not impose detailed boundaries that determine every

single aspect of practice. Researchers of a realist orientation are not prohibited from using a certain practice normally associated with qualitative inquiry and vice versa. Confining the quantitative-qualitative issue to this level allows one to support Cronbach's claim that the two approaches can be "mixed".

Yin (1984) supports this claim, when he refers specifically to case studies that can be entirely limited to quantitative evidence. Case studies should not be confused with the burgeoning use of "qualitative research". The essence of qualitative research consists of two conditions: (a) the use of close up detailed observation of the natural world by the investigator, and (b) the attempt to avoid prior commitment to any theoretical model. This type of research does not always produce case studies, nor are case studies always limited to these two conditions. Case studies can be entirely devoid of direct observation and based entirely on quantitative evidence.

However, Smith and Heshusius point out that acceptance of this point can not lead to the conclusion that the two approaches are compatible or complementary because for quantitative inquiry and for the compatibility phase of qualitative inquiry, certain sets of practices, as opposed to particular, individual ones, are thought necessary to establish major conditions of inquiry such as the validity and reliability of studies.

Qualitative inquiry does not stipulate that certain things must be done or that validity is a matter of techniques properly applied. Smith and Heshusius point out that a confusion of method as logic of justification with method as "how-to-do-it" has allowed people to draw, even if only implicitly, an erroneous conclusion. That certain individual procedures can be mixed does not mean there are no differences of consequence. If one extends the different sets of assumptions to their logical implications, it is clear that the two perspectives part company over major issues such as the conceptualization given such basic conditions as validity and reliability, the place of techniques in the inquiry process, and the interpretation of research results. (p. 9)

Rist (1982) points out that the "hegemony" of the empirical analytic approach to inquiry has dissolved as researchers have come to recognize that

there are multiple routes and multiple destinations for their efforts, and that the idea, "what cannot be measured cannot be important" (p. 439), is a fallacy. Qualitative research, in Rist's view, encompasses all of the following approaches: field work, ethnography, ethnographic research, field studies, naturalistic studies and case study methodology. Qualitative research for Rist is synonymous with situational interpretive for Aoki and McCutcheon's use of "phenomenology". In his comparison of qualitative and quantitative research, Rist maintains that qualitative research uses holistic, inductive approaches which emphasize validity in gaining an understanding of the world. Quantitative research, in contrast, issues deductive approaches which emphasize reliability. Smith and Heshusius would certainly view this as an oversimplification in that they feel that both approaches are equally concerned with reliability and validity. A discussion of the case study as a research strategy follows in the next section.

B. Case Study Research

The Case Study as a Research Strategy

In general, case studies are the preferred strategy when "how" or "why" questions are being posed, when the investigator has little control over events, and when the focus is on a contemporary phenomenon within some real-life context. (Yin, 1984) The current study has all of these characteristics and consequently lends itself for the case study strategy. Since the acceptance and credibility of a case study depends largely on the design, therefore this section will deal with all relevant aspects of a case study.

When to use case studies

The case study is but one of several ways of doing social science research. It is used in many settings, including policy, political science, public

administration, community psychology, city and regional planning research, public agencies, studying neighborhoods and dissertation research, to name a few. They are appropriate to be used when it is important to retain the holistic and meaningful characteristics of real-life events. Case studies may be used for exploratory, descriptive and explanatory studies. In general, "what" questions may be either exploratory (in which case any research strategy may be employed) or about prevalence (in which surveys or the analysis of archival records would be favored). Questions that are exploratory in nature require conducting a systematic search through or into the teachers' interpretation of the curriculum. They are essentially investigative in nature. "How" and "why" questions are likely to favor the use of case studies, experiments, or histories. (Yin, p. 19)

Extent of control over behavioral events

The case study is preferred in examining contemporary events, but when the relevant behaviors cannot be manipulated. In contrast to histories, case studies normally add two additional sources of evidence, direct observation and systematic interviewing. While there is certainly overlap between the various strategies, the case study's unique strength is the ability to deal with a full variety of evidence - documents, artifacts, interviews, surveys and observations. (Yin, p. 20) It is also possible to use more than one strategy in any given study, for example, a survey within a case study or a case study within a survey.

Traditional prejudices against the case study strategy

Yin (1984) points out that while the case study is a distinctive form of empirical inquiry, many researchers have disdain for the strategy. He cites three reasons why this is so.

1. The lack of rigor of case studies.
2. There is little basis for scientific generalization.
3. They take too long and result in massive, unreadable documents.

Walker (1983) who engages himself in case study research, identifies three common problems with case study research which differ from those general criticisms identified by Yin. The common problems being that:

1. Case study research is an intervention, and often an uncontrolled intervention, in the lives of others.
2. Case study research provides a biased view, a distorted picture of the way things are.
3. Case study research is essentially conservative.

While Yin feels that these common concerns can be allayed and that they have been misdirected, he admits that good case studies are very difficult to do. With regard to the concern about generalizability, he suggests that case studies, like experiments, are generalizable to theoretical propositions and not to populations or universes (p. 21).

Definition of the case study as a research strategy

Previous definitions of case studies have merely repeated the types of topics to which case studies have been applied. This is largely responsible for the confusion created about them.

Yin (1984), in citing from his earlier work, offers the following technical definition of case studies. A case study is an empirical inquiry that:

- * investigates a contemporary phenomenon within its real-life context; when
- * the boundaries between phenomenon and context are not clearly evident; and in which
- * multiple sources of evidence are used. (p. 23)

The definition not only helps us understand case studies, but it also distinguishes them from other research strategies. Given this definition, the following statements can be made:

1. Case study research includes both single and multiple-case studies.
2. Case studies can include, and even be limited to, quantitative evidence.
3. Case studies are not synonymous with qualitative research.
4. Case studies can be based entirely on quantitative evidence.
5. Case studies need not always include direct detailed observations as a source of evidence.
6. Case studies have a distinctive place in evaluation research.

In relation to case studies having a distinctive place in evaluation research, there are at least four different applications:

1. To explain the causal links in real-life interventions that are too complex for the survey or experimental strategy.
2. To describe the real-life context in which an intervention has occurred.
3. An evaluation can benefit, again in a descriptive mode, from an illustrative case study of the intervention itself.
4. To explore those situations in which the intervention being evaluated has no clear, single set of outcomes. (p. 25)

Designing Case Studies

A research design is the logic that links the data to be collected (and the conclusions to be drawn) to the initial questions of a study. For case studies, four major types of designs are relevant, which may be viewed in terms of a 2 x 2 matrix, with the first pair of categories consisting of single-case and multiple-case designs and the second pair, which is based on the unit or units of analysis to be covered, consisting of holistic and embedded designs.

In selecting any of these designs, the researcher must also maximize four aspects of the quality of any design: (1) construct validity, (2) internal validity (for explanatory or causal case studies only), (3) external validity, and (4) reliability. (Yin, 1984, p. 27)

Definition of Research Designs

A research design is an active plan for getting from an initial set of questions to a set of conclusions about these questions. Yin (1984) clearly identifies with the definition provided by Nachmias and Nachmias (1976) who describe a research design as a plan that:

...guides the investigator in the process of collecting, analyzing, and interpreting observations. It is a logical method of proof that allows the researcher to draw inferences concerning causal relations among the variables under investigation. The research design also defines the domain of generalizability, that is, whether the obtained interpretations can be generalized to a larger population or to different situations.

Another way of thinking about a research design is as a "blueprint" of research dealing with at least four problems: what questions to study, what data are relevant, what data to collect, and how to analyze the results. (Philliber, Schwab & Samsloss, 1980)

Components of Research Designs

Yin (1984) identifies five components of a research design that are important:

1. a study's questions;
2. its propositions, if any;
3. its unit(s) of analysis;
4. the logic linking the data to the propositions; and
5. the criteria for interpreting the findings.

The study's questions in terms of the form and nature have already been dealt with on the previous pages.

Study propositions direct attention to areas of the research that should be examined within the scope of study. The absence of propositions might lead to the collection of too many data or data not relevant to the study. The unit of analysis is related to the fundamental problem of defining what the "case" is.

Linking the data to propositions and determining the criteria for

interpreting the findings represent the data analysis steps in case study research. While these two components are not well defined yet, there are some approaches that can be used for case studies. One such approach is "pattern-matching", where several pieces of information from the same case may be related to some theoretical proposition. However, the question of whether or not the data match one pattern better than the other is still a difficult question to be answered, because there is no precise way of setting the criteria for interpreting the findings of case studies, unless the different patterns are sufficiently contrasting in relation to two rival propositions.

Criteria for judging the quality of research design

The four quality tests for case studies are discussed in the following section of this dissertation. This section will deal with the case study tactics applicable to the four quality tests and the current research study.

Yin (1984) identifies three case study tactics in relation to the quality test of construct validity. The first one is the use of multiple sources of evidence, which can involve such things as classroom observations, interviews, questionnaire surveys and tests. Second, establish chain of evidence. This tactic implies that an external observer should be able to trace the steps from the initial research questions to the ultimate case study conclusions and from the conclusions to the initial research questions. Third, the draft case study report should be reviewed by the key informants. Schatzman and Strauss (1973, p. 134) point out that this tactic is a way of corroborating the essential facts and evidence presented in the case study report.

The case study tactics in relation to the quality test of internal validity are pattern matching, explanation building and time series analysis. The descriptive approach is often used (Yin, 1984) to quantify events which are then used in a causal sense to explain a relationship. Pattern-matching is a frequently used strategy which is based on the data patterns found and it involves a comparisons to predicted patterns. This helps a case study to

strengthen its internal validity, particularly when the patterns coincide.

Other modes of analysis are used in case studies as well that have the effect of strengthening internal validity. In particular, this often includes an analysis of embedded units within the case study. Another analysis that strengthens internal validity is "Repeated Observation" across different cases.

The third quality test is external validity. While in the broadest sense it means establishing the domain to which a study's findings can be generalized, the analogy to samples and universes is incorrect when dealing with case studies. Consequently, the external validity problem has been a major barrier in doing case studies. Critics are implicitly contrasting case studies to survey research, where a correctly selected sample readily generalizes to a larger universe. (Yin, 1984, p. 39) Survey research relies on statistical generalization, whereas case studies, like experiments, rely on analytical generalizations.

In analytical generalizations, the researcher is striving to generalize a particular set of results to some broader theory, which in turn is tested through the replication of the findings in subsequent case studies. If the theory is correct or it has specified certain outcomes or results, then the outcomes or results should occur in subsequent case studies.

The fourth quality test is reliability, which concerns itself with minimizing the errors and biases in a study. This suggests that if another researcher were to follow exactly the same procedures as described in a given study, the same results and conclusions should be achieved. This calls for rigid adherence to two specific tactics in case study research: (1) the use of case study protocol and (2) the development of a case study data base. (Schatzman and Strauss, 1973; Yin, 1984) The general way of approaching the reliability problem is to make as many steps as possible as operational as possible.

Case study designs

Yin (1984) identifies four basic types of designs for case studies. He distinguishes between the single and multiple-case studies and within these

two types he identified two types of analysis - unitary or multiple units of analysis.

The single-case study design is analogous to a single experiment and is justified in a number of circumstances. One rationale is when it represents the critical case in testing a well formulated theory. In this situation it may represent a significant contribution to knowledge and theory-building. The second rationale for conducting single-case studies is where the case represents an extreme or unique case. This is often found in medical research. A third rationale for a single-case study is the revelatory case. This situation exists when a researcher has an opportunity to observe and analyze a phenomenon previously inaccessible to scientific investigation.

A single-case study may involve more than one unit of analysis. This occurs when, within a single case, attention is also given to a subunit or subunits. A school evaluation is an example of the school being the case, but the analysis might also include outcomes from a variety of subprograms. Such a design would be called an embedded case study design in contrast to a holistic design or single unit of analysis. The holistic design is advantageous when no logical subunits can be identified or when the relevant theory underlying the case study is itself of a holistic nature. However, potential problems arise, when the global approach allows a researcher to avoid examining any specific phenomenon in operational detail.

A further problem with the holistic design is that the entire nature of the case study may shift, unbeknownst to the researcher, during the course of study. In order to increase the sensitivity to such slippage, the researcher should have a set of subunits. The embedded design can serve as an important device for focusing a case study inquiry. However, the embedded design also has potential problems. A major one occurs when the case study focuses only on the subunit level and fails to return to the larger unit of analysis. When a study contains more than a single case, the study has to use a multiple-case design.

Yin (1984) does not differentiate between single and multiple-case studies from a methodological point of view. He states that the choice between

single and multiple-case designs remains within the same methodological framework. The choice between the two is considered one of research design, with both being included under the case study strategy. Multiple-case designs have distinct advantages and disadvantages in comparison to single-case designs. The evidence from multiple-case designs is often considered more compelling. However, while not always applicable, the multiple-case design often requires extensive resources and time which may be well beyond the means of a single researcher. Therefore, when one undertakes a multiple-case study, every case should serve a specific purpose within the overall study. Yin points out that the multiple cases have incorrectly been considered similar to the multiple respondents in a survey or to the multiple subjects within an experiment - that is, to follow a "sampling" logic, when, in fact, multiple cases follow a "replication" logic. The methodological differences between these two views are revealed by the different rationales underlying each logic. The replication logic is analogous to that used in multiple experiments. For a research study involving several cases, replication is said to have taken place if the results obtained are similar for two or more cases. In each of these situations, an individual case stands alone and the analysis must follow cross-experiment rather than within experiment design and logic. An important step in all of these replication procedures is the development of a rich, theoretical framework. (Yin, 1984, p. 49) The framework needs to state the conditions under which a particular phenomenon is likely to be found (a literal replication) as well as the conditions when it is not likely to be found (a theoretical replication). The theoretical framework later becomes the vehicle for generalizing to new cases. Furthermore, just as with experimental science, if some of the empirical cases do not work as predicted, modification must be made to the theory.

With regard to the question as to whether or not multiple-case studies should be holistic or embedded, Yin argues that they can be either. The type of design being selected depends on the phenomenon being studied. It is important to note that the data are not pooled across the cases, rather, they are

part of the findings for each individual case. The pooling of survey data across the cases changes the multiple-case study design to a survey study design. However, it is appropriate within the multiple-case study design to conduct a cross-case analysis. When an embedded design is used, each individual case study may in fact include the collection and analysis of highly quantitative data, including the use of surveys within each case.

Preparing for Data Collection

Although conducting a case study begins with the definition of the problems or issues to be studied and the development of a case study design, the preparation for data collection is an extremely important aspect of the entire process. Yin (1984) suggests that the preparation for data collection should include consideration of the skills of the investigator, the training and preparation for the specific case study, the development of a case study protocol, and the conduct of a pilot case study. While all of these components are important, the development of a protocol is an effective way of dealing with the overall problem of increasing the reliability of case studies. With regard to the prerequisite skills of a case study investigator, Yin identified the following list of commonly required skills: (1) the investigator should be able to ask good questions, (2) be a good listener, (3) be adaptive and flexible, (4) have a firm grasp of the issues being studied, and (5) be unbiased by preconceived notions. (p. 56)

The Case Study Protocol

The protocol is a major tactic in increasing the reliability of case study research and is intended to guide the investigator in carrying out the case study. (Yin, 1984, p. 64) A case study protocol is more than an instrument. It contains the instrument as well as the procedures and general rules that are to be

followed in using the instrument. Case study protocols are important to have for all case studies but particularly so when using a multiple-case design.

According to Yin (1984), a case study protocol should have the following sections:

1. Overview of the case study project, which includes the project objectives, case study issues and relevant readings about the topic being investigated.
2. Field procedures, which include the credentials and access to the case study "sites", general sources of information and procedural reminders.
3. Case study questions, which include specific questions that the investigator must keep in mind in collecting data, "table shells" for specific arrays of data, and the potential sources of information for answering each question. Although the research questions are the central focus of the study, they are rarely posed directly to the respondents. The research questions, in essence, serve the researcher as a reminder regarding the information that needs to be collected, and why. Often the instruments used for data collection include questions that are answerable at the single-case level. Questions asked of the findings across multiple cases, beyond multiple cases or beyond the scope of the study are not asked of individual cases. However, responses received or data derived from the individual cases are used to make inferences or provide answers to the questions beyond the single case level.
4. Guide for the case study report, which should include an outline, format for the narrative and specification of any bibliographical information and other documentation.

The pilot case study

Conducting a pilot case study is seen as an important step in the preparation for data collection. (Yin, 1984) He views the pilot case study as a way of helping the investigator in refining the data collection plans with respect to both the content of the data and the procedures to be followed. Furthermore, it serves to increase the validity and reliability of the instruments and procedures used.

Collecting the Evidence

Yin (1984) indicates that the evidence for case studies may come from six sources: documents, archival records, interviews, direct observation, participant-observation and physical artifacts. Each of these sources requires slightly different skills and methodological procedures. He adds that in addition to the attention given to these individual sources, some overriding principles are important to any data collection effort in doing case studies. These include the use of: (1) multiple sources of evidence, converging on the same set of facts or findings; (2) a case study data base, meaning the assembly of evidence distinct from the final case study report, and (3) a chain of evidence, that is, explicit links between the questions asked, the data collected, and the conclusions drawn.

An important source of evidence is the interview. There are different types of interviews that can be used, the open-ended interview and the focused interview which follows a predetermined set of interview questions.

Direct observation or classroom observations serve as another important source of evidence in case studies. Participant-observation is a special mode of observation in which the researcher is not merely a passive observer but an active participant. This mode of observation provides a number of unusual opportunities for collecting case study data, but it also involves major problems. Yin (1984) indicates that the most distinctive advantage relates to the opportunity to gain access to events or groups, but is quick to point out that the major problem relates to the potential bias produced by the participant-observer.

Principles of data collection

The research literature identifies three important principles that need to be adhered to when collecting data. These are: (1) Using multiple sources of evidence, (2) Creating a case study data base and (3) Maintaining a chain of evidence. Yin (1984) suggests that when these principles are properly applied,

they can assist considerably in establishing construct validity and reliability of a case study.

The second principle, creating a case study data base, refers specifically to a formal, retrievable set of data that other researchers can review. Such evidentiary base increases markedly the reliability of a case study. (Yin, 1984) It is also essential that the case study report or dissertation contain sufficient data so that the reader can draw independent conclusions about the case study.

The third principle, maintaining a chain of evidence, also increases the reliability of the information in a case study. A study that adheres to this principle would allow the derivation of any evidence from initial research questions to ultimate case study conclusions.

Analyzing Case Study Evidence

Analyzing case study evidence is especially difficult because the strategies and techniques have not been well defined in the past. (Yin, 1984, p. 99) However, when conducting case study research, it is essential that a general analytic strategy be developed prior to embarking on the data collection process. The research literature identifies three dominant analytic techniques which are applicable for single or multiple-case designs.

The use of statistical analyses is often not applicable at the level of the whole case because of the small number of cases. However, this does not preclude the use of statistical analyses when one has an embedded unit of analysis within a case study. Yin (1984) points out that the ultimate goal of the analysis is to treat the evidence fairly, to produce compelling analytic conclusions and to rule out alternative interpretations. He describes two general strategies which guide in the selection of specific techniques. The first and more preferred strategy follows theoretical propositions that led to the case study. It is assumed that the original objectives and design of the case study are based on such propositions which in turn reflect a set of research questions.

The propositions, therefore, shape the data collection plan and the application of relevant analytic strategies. Propositions also help to organize the entire case study and to define alternative explanations to be examined. Theoretical propositions can be very useful in guiding case study analysis, particularly where they reflect causal relations.

A second general analytic strategy is to develop a descriptive framework for organizing the case study. This is particularly useful when theoretical propositions are absent. The descriptive approach may also help in identifying appropriate causal links to be analyzed. This approach does not preclude the quantification of events through enumeration or tabulation.

In identifying specific ways of conducting case study analysis, Yin (1984) suggests three dominant modes of analysis. Pattern-matching is viewed as one of the most desirable strategies for case-study analysis. The logic behind pattern-matching is that an empirically based pattern is compared with a predicted one or with several alternative predictions. If the patterns coincide, the results can help a case study to strengthen its internal validity. If the case study is a descriptive one, pattern-matching is still relevant as long as the predicted pattern of specific variables is defined prior to data collection.

Yin differentiates between the nonequivalent dependent variables pattern and pattern-matching of independent variables. The former would imply a variety of outcomes for each dependent variable and the latter would focus on how and why an outcome occurred in each case. Pattern-matching of independent variables requires the development of rival theoretical propositions articulated in operational terms. The important characteristic of these rival explanations is that each involves a pattern of independent variables that is mutually exclusive. (Yin 1984, p. 105) This implies if one explanation is to be valid, the others cannot be. Pattern-matching of independent variables can be done with either a single or multiple cases.

Whatever technique is being employed, the fundamental comparison between the predicted and the actual pattern may not involve quantitative or statistical criteria. It is therefore important that the techniques employed in the

analysis are sensitive enough to uncover gross matches or mismatches and ultimately valid conclusions.

Explanation-building is another analytic strategy and is a special type of pattern-matching. Here, the goal is to analyze the case study data by building an explanation about the case. To "explain" a phenomenon is to stipulate a set of causal links about it. The causal links are similar to the independent variables in the previously described use of rival explanations. Most frequently case studies are using the narrative form for explanation-building. The preferred way is to have explanations reflect some theoretically significant propositions. Explanation-building is iterative in nature and is similar to the process of refining a set of ideas. When applied to multiple-case studies, the result of the explanation-building process is also the creation of a cross-case analysis, not simply an analysis of each individual case.

A third analytic strategy is described as time-series analysis which is analogous to the time-series analysis conducted in experiments and quasi-experiments. Yin (1984) identifies three types of time-series analyses: simple, complex and chronologies. He says, that the essential logic underlying a time-series design is the match between a trend of data points compared to (a) a theoretically significant trend specified before the onset of the investigation, versus (b) some rival trend, also specified earlier, versus (c) any trend based on some artifact or threat to internal validity. When time-series analysis is used in a case study, it is important to identify the specific indicator(s) to be traced over time, as well as the specific time intervals to be covered. Whatever the stipulated nature of the time-series, the important case study objective is to examine some relevant "how" and "why" questions about the relationship of events over time, not merely to observe the time trends alone.

Lesser modes of analysis

In addition to the dominant modes of analysis identified in the previous section, three lesser modes of analysis may be used in case studies:

(1) analysis of embedded units of analysis, (2) repeated observation, and (3) the case survey approach. However, these lesser modes of analysis must be used in combination with one of the dominant modes in order to produce a full case study analysis.

When a case study design includes an embedded unit of analysis - that is, a lesser unit than the case itself, for which numerous data points have been collected - the relevant analytic approach can cover nearly any of the techniques used in research. For each analysis, the pertinent analytic strategy reflects the proposition to be examined for the embedded unit. These propositions are related to but different from the propositions for the larger case. The larger case is the major interest of the study. If the embedded unit is itself the main focus of attention and the larger case serves merely as contextual matter, the study ceases to be a case study. Yin (1984) emphasizes this point when he says that this distinction appears most clearly in multiple, embedded case studies, in which it is appropriate that the analysis of the embedded unit of analysis must first be conducted within each case. (p. 115) The results are interpreted at the single-case level and may be treated as factors in a pattern-matching or explanation-building analysis at the single-case level. The patterns or explanations for each single case may then be compared across cases, following the replication mode for multiple cases. The conclusions drawn for the multiple cases can become the conclusions for the overall case study.

The use of repeated observations is also considered a lesser mode of analysis and it is used in conjunction with one of the dominant modes of analysis. Repeated observations as a mode of analysis is frequently used in case studies, where the repetition is occurring across different teachers, students and classrooms.

The third of the lesser modes of analysis is doing a case survey. This mode of analysis is only used when there are many case studies in the area under investigation. Yin suggests that the case survey is a relevant technique when the research objective is explicitly that of a secondary analysis - for

example, to determine "What the existing literature says" about a certain topic. However, the case survey is not considered a dominant mode of analysis for designing and doing a fresh set of case studies. (p. 118)

Composing the Case Study Report

Yin (1984) states that reporting of a case study can take written or oral forms. However, regardless of the form similar steps need to be followed in the compositional process: identifying the audience for the report, developing the compositional structure and following certain procedures, such as having it reviewed by informed persons.

With regard to the compositional structure, Yin suggests six alternatives: linear-analytic, comparative, chronological, theory-building, "suspense", and unsequenced structures:

Varieties of written reports

Yin (1984) suggests that there are at least four important varieties: (1) the classic single-case study, (2) the multiple-case version of this classic single case, (3) the multiple or single-case study without traditional narrative, and (4) the multiple-case study.

The classic single-case study uses a single narrative to describe and analyze the case and it may appear as a book, a report, or a journal article. It may also be augmented with graphical, pictorial or tabular information.

The multiple-case version of this classic single-case contains multiple narratives, often in separate chapters, about each of the cases singly. In addition, the report contains a chapter covering the cross-case analysis and results. If the cross-case analysis is lengthy, individual cases may also be presented as part of the appendix.

The third type of reporting covers either a multiple - or a single-case study but does not contain the traditional narrative. Here, the composition for

each case follows a series of questions and answers based on the case study data base. This format avoids the problem of lengthy writing and the investigator proceeds immediately to answer the required set of questions.

The multiple-case report is the fourth type of a report. There may be no separate chapters devoted to individual cases. Rather, the entire report may consist of the cross-case analysis, whether purely descriptive or covering explanatory topics. Each chapter would be devoted to a separate cross-case issue, and the information from the individual cases would be dispersed throughout each chapter. Summary information about the individual cases is often ignored altogether or presented in abbreviated vignettes.

Compositional structures for case studies

Depending on the different purpose of case studies, six different compositional structures may be used to write the final report:

Linear-analytic structures: This is a standard approach for composing research reports and the literature suggests that this structure is most comfortable to investigators and advantageous to dissertation committees. This structure is applicable to explanatory, descriptive or exploratory case studies of single- or multiple-case design.

Comparative structures: In this structure the case is repeated several times, comparing alternative descriptions or explanations of the same case. The purpose of the repetition is to show the degree to which the facts fit each model. The repetitions actually illustrate a pattern-matching technique at work.

Chronological structures: The case study evidence is presented in chronological order. This structure is employed in doing explanatory case studies, because causal sequences must occur linearly over time.

Theory-building structures: In this structure, the sequence of chapters follows some theory-building logic. Each chapter reveals a new part of the theoretical argument being made. This approach is relevant to both explanatory and exploratory case studies. Explanatory cases examine the various facets of a causal argument and exploratory cases debate the value of

further investigating various hypotheses or propositions.

Suspense structures: The results or outcomes of a case study are presented in the first chapter and the remainder of the case study is then devoted to the explanation of this outcome. Additional chapters may also deal with alternative explanations. This approach is particularly relevant to explanatory case studies.

Unsequenced structures: The sequence or structure of the chapters is of no particular importance. Descriptive case studies of an organization often employ this structure. It is important, when using this structure, to test for completeness, otherwise the investigator may be accused of being biased.

Procedures in doing a case study report

Yin (1984) identifies three important procedures when doing a case study report. These procedures refer to : (1) general tactic for starting a composition, (2) covering the problem of whether to leave the case identities anonymous, and (3) reviewing procedures for increasing the construct validity of a case study.

With regard to the first procedure, it is important to start composing early in the analytic process. Following the literature review, the bibliography and the methodological sections should be drafted. From the methodological section major procedures for data collection and analysis are derived which become part of the case study design. Following the data collection and prior to analysis, composition of the descriptive data about the cases should be completed. This includes the qualitative and quantitative data about the cases. At this point the researcher is in a position to make choices regarding the type of composition to be used and the type of structure to be followed.

The second procedure presents the researcher with a choice regarding the anonymity of the case. The anonymity issue can be raised at the case level and the individual level, however, it can also involve both, the case and the individuals. While anonymity is not considered a desirable outcome, it is often

necessary, particularly when the subjects have been assured of that.

The third procedure deals with the validating process. This procedure suggests that the draft report be reviewed by peers and the participants in the case. Schatzman and Strauss (1973, p. 134) consider such a review not just professional courtesy, but they also identify such a procedure as a way of corroborating the essential facts and evidence presented in the case report. Reviewers may well disagree with the conclusions and interpretations, but they should not disagree with the actual facts of the case. Yin (1984) makes the point that the corrections made through this process will enhance the accuracy of the case study, hence increasing the construct validity of the study.

In discussing the question of what makes an exemplary case study, Yin (1984) identifies five general characteristics. These are: (1) the case must be significant, (2) the case study must be "complete", (3) the case study must consider alternative perspectives, (4) the case must display sufficient evidence and (5) the case study must be composed in an engaging manner.

The significance of the study may be related to its general interest to practitioners or its potential for both discovery and theory development.

The "completeness" of the study is another characteristics of an exemplary study. It asks if the phenomenon under study and its context are given explicit attention through the use of multiple strategies. This often involves a variety of instruments, resulting in multiple sources of relevant evidence. A third factor in making a study significant is by avoiding certain artificial conditions such as resource limitations and time.

Considering alternative perspectives is another major prerequisite for an exemplary case study. It is important that the point of view of various major actors in the case is being incorporated.

The case study must also display sufficient evidence in order to be exemplary. This means that the critical pieces of evidence are contained within the case study report, that the evidence is presented neutrally, that the researcher knows his subject, and that there is a chain of evidence.

The final characteristic of an exemplary case study is that the study must

be composed in an engaging manner. Yin (1984) states that the report must be written in a clear writing style, it must entice the reader and seduce the eye and above all, it must radiate enthusiasm about the investigation and its conclusions.

C. Validity and Reliability In Qualitative and Quantitative Research

The concepts of validity and reliability are of central importance for both research orientations. While both concepts have been well established in the context of quantitative research, their meaning in the context of qualitative inquiry is still in the initial stages. The meaning of the terms, validity and reliability will be explored in the context of both quantitative and qualitative research in what follows below.

With regard to any research orientation, the first and foremost question to be asked with respect to any testing procedure or research instrument is: How valid is it? When one asks this question, one is really inquiring whether the test measures what one wants it to measure, all of what one wants it to measure, and nothing but what one wants it to measure.

In quantitative research, validity is defined as, "the extent to which a test measures what we actually wish to measure", while reliability refers to, "the accuracy of the measure whatever it does measure" (Thorndike & Hagen, 1969, p. 163 and p. 177, respectively). It seems that validity is of primary importance, since we must establish first that we are measuring what we wish to measure before establishing whether our measuring device gives consistent and accurate value. Reliability, although an important quality in a measuring device, cannot supplant validity, since the measuring device may consistently and accurately measure behavior other than that intended. Campbell and Stanley (1963) make a distinction between internal and external validity. Internal validity is the basic minimum without which any experiment is uninterpretable. It refers to correctly establishing a causal relationship whereby certain conditions are shown to lead to other conditions as distinguished from spurious

relationships.

Internal validity is a concern only for causal or explanatory studies, where an investigator is trying to determine whether event x led to event y . The concern over internal validity, for case study research, may be extended to the broader problem of making inferences. The use of various analytic tactics, such as pattern-matching, explanation-building and time-series analysis may assist in overcoming this problem. (Yin, 1984, p. 38)

External validity refers to the generalizability of the findings to other populations, settings, treatment variables and measurement variables. Yin points out that the external validity problem has been a major barrier in doing case studies. Critics typically focus on the lack of generalizability that a single case offers, however, this analogy to samples and universes is incorrect when dealing with case studies. Survey research relies on statistical generalization, whereas case studies rely on analytical generalization. In analytical generalization, the investigator is striving to generalize a particular set of results to some broader theory. However, generalization in case studies is not automatic. A particular theory must be tested through replication of the findings in other settings. Bracht and Glass (1968) have further expanded the concept of external validity by developing the notions of population validity and ecological validity. Population validity is concerned with determining what population of subjects can be expected to behave in the same way as did the subjects under consideration. Ecological validity refers to the conditions under which the same results can be expected. There are some kinds of validity that are not considered quantifiable while others lend themselves to numerical estimation. Ebel (1972) has classified these two types of validity as being direct and indirect, respectively, while Thorndike and Hagen (1969) refer to them as being dependent on professional judgement and rational analysis. Face validity and content validity require professional judgement, while concurrent, predictive and construct validity are amenable to rational analysis or numerical estimation.

Face validity has been defined as the reasonableness or plausibility of test tasks, from the point of view of the person being tested in terms of what he

considers that the test is measuring (Thorndike & Hagen, 1969), or "what the test appears to measure". (Ebel, 1972) Thorndike & Hagen emphasize that what a test looks like may be of importance in determining its acceptability to those writing it. Content validity refers to the faithfulness with which the test represents or reproduces an area of knowledge and it rests essentially on authorities' judgement of the representativeness of the sampling of the universe in which the researcher is interested. This implies that the universe is clearly defined and each item selected is an appropriate representative of the universe and the property being measured. Ebel (1972) insists that the directions for administering and scoring the test should be subjected to logical analysis and expert judgement, as should the rationale and specifications for the instrument. Content validity can be assumed only after all these judgmental assurances of validity have been received. (Ebel, 1972)

Concurrent and predictive validity are two measures of validity which are estimated empirically. Both of these are established by the degree of consistency which a test has with a criterion measure, usually expressed by a correlation coefficient. (Cronbach & Miehle, 1967) Concurrent and predictive validity differ from each other primarily on the basis of time. Concurrent validity is established by comparison of the instrument with the criterion at the same time, while predictive validity is established by comparison of the instrument with a criterion measure in the future. Thorndike and Hagen (1969) identified four desirable qualities in a criterion measure. In order of importance they are: (1) relevancy, (2) freedom from bias, (3) reliability, and (4) availability. With regard to relevancy, it is necessary to rely on professional judgement rather than empirical evidence to provide the appraisal of the degree to which any partial criterion measure is relevant to the ultimate criterion. A second factor important in a criterion measure is that of freedom from bias. This means that the measure should provide each person with the same opportunity to make a good score. The third factor is that of reliability. Here the requirement is that it is stable or reproducible. Finally, in the choice of criterion measure one always encounters practical problems of convenience and availability. Any choice of a

criterion measure must take this practical limit into account.

Construct validity is a third measure of validity which refers to the accuracy with which the test describes an individual in terms of some psychological trait or construct and which can be estimated empirically. (Thorndike and Hagen, 1969) The measured construct is taken to be some postulated attribute which is assumed to be reflected in test performance. Construct validation is used when no criterion measure of a construct exists. For example, in the current study the construct validity of the "Classroom Practices of Trigonometry Teachers Survey" could be estimated by factor analysis of the teachers' responses to those items which involve using Likert rating scales. However, the number of teachers involved would render such factor analysis meaningless. This implies that the instruments used in the current study would not be validated in a numerical sense and therefore other validation procedures need to be employed.

Yin (1984) refers to construct validity as establishing correct operational measures for the concepts being studied. (p.36) Construct validity is especially problematic in case study research. The criticism against case study research has often centered around the lack of an operational set of measures. In order to meet the test of construct validity, Yin maintains that an investigator must be sure to cover two steps: (1) select the specific types of changes that are to be studied (in relation to the original objectives of the study) and (2) demonstrate that the selected measures of these changes do indeed reflect the specific types of change that have been selected. Three factors are available to increase construct validity. First, use of multiple sources of evidence, in a manner encouraging convergent lines of inquiry. This tactic is relevant during data collection. Second, establish a chain of evidence, which is also relevant during data collection. Third, have the draft case study reviewed by key informants.

While it is recognized that some measures of validity can be estimated empirically and that there is greater acceptance and comfort with rational analysis, the numerical values produced still require a great deal of subjective judgement on the part of the researcher in interpreting and assessing their

meaning for a particular purpose. With regard to quantitative research there are well established guidelines for interpreting numerical assessment of validity. It can be argued then that subjective judgement is an essential, even unavoidable, aspect of assessing the validity of both qualitative and quantitative research.

With regard to validity of qualitative inquiry, Dawson (1979) suggests that getting the subjects to confirm and disconfirm the observations is a direct and effective way of establishing validity of an observation. Furthermore, it permits the researcher, through discussion with the subjects involved, to derive a fuller meaning of the event, phenomenon or situation. In response to the concern about generalizability of case studies, Stake (1978) offers the following comment:

...I claim that case studies will often be the preferred method of research because they may be epistemologically in harmony with the reader's experience and thus to that person a natural basis for generalization. (p.5)

He further states that the appropriateness of the case study approach depends on the purpose of the research.

When explanation, propositional knowledge, and law are the aims of an inquiry, the case study will often be at a disadvantage. When the aims are understanding, extension of experience, and increase in conviction in that which is known, the disadvantage disappears. (p.6)

The case study is one of several ways of doing social science research. In his book, Case Study Research-Design and Methods, Yin (1984) states that each strategy has peculiar advantages and disadvantages, depending upon three conditions: (1) the type of research question, (2) the control an investigator has over actual behavioral events, and (3) the focus on contemporary as opposed to historical phenomena. He also states that case studies are the preferred strategy when "how" or "why" questions are being posed and when the phenomenon to be studied occurs within some real-life context.

In the context of qualitative research, Dawson (1979) has conceptualized validity as, "the adequacy of a description as a representation of a social situation". (p.1) He points out that since "adequacy" implies purpose, validity is associated with purpose or utility in this conceptualization. Valid knowledge is taken to be knowledge which is of value for a given purpose. A typical study often has multiple purposes, therefore, it will have a different validity for different aspects of the study. While Dawson's conceptualization of validity within qualitative inquiry does not permit rational analysis, it is consistent with Cronbach's (1981) views of the validity of test instruments.

One validates, not a test, but an interpretation of data arising from a specified procedure. A single instrument is used in many different ways - Smith's reading test may be used to screen applicants for professional training, to plan remedial instruction in reading, to measure the effectiveness of an instructional program, etc. Since each application is based on a different interpretation, the evidence that justifies one application may have little relevance to the next. Because every interpretation has its own degree of validity, one can never reach the simple conclusion that a particular test "is valid". (p.447, emphasis in the original)

Validity concerns within qualitative research are often addressed through developing strategies that confirm or disconfirm findings. Various investigators have used different approaches in applying this strategy. Becker (1969) has developed procedures for coding and analyzing field notes, by compiling frequencies of the observation of the phenomenon. The observations are categorized according to whether the observations were statements or activities and according to the circumstances under which they were made. This allows both the researcher and research audience to judge the amount of confidence they have in the research conclusions. Douglas (1976) has developed an approach which he calls "investigative social research" which takes the basic position that people are evasive or dishonest with researchers. In his opinion, information gathered should be continuously "tested out" using the following approach:

1. Comparing a supposed fact, member account, etc. with the most reliable ideas and generally patterned facts that the researcher has from his prior experience.
2. Comparing one's own ideas and inferences with the observed facts in a setting.

The confirmation strategy has been used by many qualitative researchers. It involves the participants in the study to react to the investigator's perceptions and interpretation, confirming or disconfirming them. (Yin, 1984) Participant perceptions and comments are not necessarily more accurate than those of the researcher, but Dawson (1979) points out that those data whose validity is being challenged should be examined carefully.

Triangulation, in which data bearing on an issue is gathered from multiple sources, is frequently used by qualitative researchers. (Dawson, 1979) The study conducted by Stake and Easley (1978) incorporated a range of methods, but difficulties were experienced in preserving some features of the original design. This was partly due to the reluctance of their observers to utilize certain more structured data gathering techniques.

In their book, Field Research, Schatzman and Strauss (1973) have suggested a number of strategies to enhance internal validity in qualitative research. Among others, they emphasize the importance of researcher-participant relations. Schatzman and Strauss are suggesting that the confidence and trust between the researcher and participant is the best guarantee of spontaneity, and hence validity.

In addition to the variety of validity concepts found in both qualitative and quantitative research literature, Brinberg and McGrath (1982) state that there are three underlying meanings in the concept of validity. Validity may be viewed as value, as correspondence and as robustness. Validity as value is principally as a pre-study concern. It includes deciding what elements and relations to sample from the substantive domain or what aspects of that domain will be regarded as real and important events and processes. Similarly, for the

methodological domain and for the conceptual domain, sampling implies deciding what elements and relations will be regarded as important and legitimate. Brinberg and McGrath argue that in all three cases, applying criteria and standards to decide what elements and relations are regarded as real implies applying a set of values to such selections. They view these values as validity-like, but they are neither correspondence or robustness.

Validity as correspondence is particularly important in measures of internal validity where correspondence between two sets of things is considered. For example, a research study may involve a set of concepts and a set of observations or two sets of measures of a single phenomenon. The correspondence notion is of particular importance during the research process where construct and predictive validity are of concern to the researcher.

Validity as robustness is of central importance to the various forms of external validity, such as ecological and population validity, which are considered after the research process. Brinberg and McGrath (1982) regard explanatory validity as an aspect of external validity. A body of research data is built and then explained and the investigator faces a key problem of having to explain how the chosen concepts account for the study findings. In addition, the investigator must explore whether there are alternative, equally plausible, conceptual explanations for the findings.

The second major question with respect to a testing instrument or measuring procedure is: How reliable is it? The researcher is now asking how accurately it measures whatever it does measure. What is the precision of the resulting score? How accurately will it be reproduced if the individual, event or phenomenon is measured again? Yin (1984) maintains that the reliability test for any study, including the case study, must demonstrate that the operation of a study, such as the data collection procedures, can be repeated with the same results. While validity is indicated by agreement among independent methods of assessment, reliability is indicated by agreement among similar methods of assessment separated by time and location. Thorndike and Hagen (1969) identified a number of methods of obtaining repeated measures estimates,

which include: (1) repetition of the same test or measure, (2) administration of a second "equivalent" form of the test, (3) subdivision of the test into two or more equivalent fractions, (4) averaging all possible split-halves, and (5) rating performances by two or more judges. Both the formula for and the method of establishing reliability are dependent upon the nature of the test and the research design.

Yin (1984) suggests that the goal of reliability is to minimize the errors and biases in a study. (p. 40) He further contends that the case study investigator must maximize the reliability to a point if a later investigator followed exactly the same procedures as described by an earlier investigator and conducted the same case study all over again, the later investigator should arrive at the same findings and conclusions. This clearly calls for the need to document the procedures followed in the case study in a meticulous way. Yin maintains that the general way of approaching the reliability problem is to make as many steps as possible as operational as possible, and to conduct research as if someone were always looking over your shoulder.

The protocol is a major tactic in increasing the reliability of case study research and it is intended to guide the investigator in carrying out the case study. The protocol should have the following sections:

- * overview of the case study project,
- * field procedures,
- * case study questions, and
- * guide for the case study report. (Yin, 1984, p. 64)

In relation to the data base, every case study project should strive to develop a formal, retrievable data base, so that in principle, other investigators can review the evidence directly and not be limited to the written report. In this manner, the data base will increase markedly the reliability of the entire case study. (Yin, p. 92) He suggests that another principle to be followed, to increase the reliability of the information in a case study, is to maintain a chain of evidence. This would allow the reader of the case study to follow the derivation

of any evidence from initial research questions to ultimate case study conclusions. If these objectives are achieved, Yin suggests, a case study also will have addressed the methodological problem of determining construct validity, thereby increasing the overall quality of the case. (p. 96)

Deutscher (1966) points out that although interviewer and observer reliability are important concerns in social science research, some other types of reliability are not applicable. He refers to the notion of test-retest or any other reliability measure involving a time sequence. He contends that they are antithetical to social science since it must make the incorrect assumption that human thought or behavior is static and, therefore, that any change in response is a reflection of either instrument error or deception. In fact, such recorded changes are more likely to reflect shifts in attitude or behavior on the part of the respondent. (p. 241)

Quantitative educational research is clearly associated with numerical results of measurement operations and implicit in this is the assumption that it is free of subject bias on the part of the marker. However, accepting that view may not necessarily lead to a full understanding of a particular phenomenon under study. As soon as quantitative researchers utilize open-ended surveys or free response questionnaires, they rely on the ability of experts or themselves to judge the score. At this point subjective bias is introduced and the problem is a parallel one to the reliability issue in qualitative research.

Deutscher (1966) points out a problematic aspect of survey research. He states that our scientific conclusions, for the most part, are based on analysis of verbal responses to questions put by an interviewer. Those responses may be written or oral and the questions may range from forced choice to open ended, but the fact remains that what we obtain from such methods are statements of attitude, opinion, norms, values, anticipation, or recall. The assumption must be made that verbal responses reflect behavioral tendencies. This inferential jump from verbal behavior to overt behavior appears to be tenuous under some conditions. (p. 235) Based on Deutscher's statement, researchers have no reason to expect congruence between a subject's attitude and action,

regardless of the theoretical orientation of their research. In fact, he is saying that they have reason to expect discrepancies between them. This is particularly relevant in studies that make the assumption that the classroom teachers' interpretation of curriculum materials is reflected in the classroom process, actions, activities, events and in the teachers' perceptions. However, given the tenuous connection between verbal behavior and overt behavior, inferences about actual classroom practice made from responses to interview questions and questionnaire items must be made with caution. Inferences about how classroom teachers interpret curriculum materials, however, can be made validly from both of these sources of information since they give information about espoused theories. (Argyris and Schön, 1974) The direct classroom observations, on the other hand, give information about theories-in-use.

D. Research Methodologies Used in Studies of Classroom Practice

The research on curriculum and instruction implementation conducted by Fullan and Pomfret (1977) points toward a wide range of data gathering techniques. These include such methods as classroom observation, survey questionnaires, focused interviews and document analysis. From their review of the research, Fullan and Pomfret conclude that classroom observations can provide the most thorough assessment of classroom practice. There are, however, some concerns raised with regard to using classroom observers to gather data on teacher use of curriculum materials.

1. The presence of observers may affect the quality of the teacher's performance.
2. Some dimensions of implementation are more difficult to assess than others. Observation methods may get at the mechanical use of an innovation, but not necessarily at other dimensions, like understanding the philosophy of the innovation.

3. The use of observation methods is expensive because of heavy time demands and therefore not feasible if large samples are involved.

Fullan and Pomfret suggest that questionnaire surveys are considerably less expensive than observational methods and could be used as a substitute for them. There is no mention in their review that the information gathered from questionnaire surveys is quite different from that obtained from classroom observations. Argyris and Schön (1974) are quick to point out that classroom observations provide the researcher with information about theories-in-use, while responses to questionnaire surveys or interviews give the researcher information on espoused theories.

The focused interview is seen by Fullan and Pomfret as having considerable merit in determining the nature and forms of teacher use of curriculum materials. Interviews, while they may be open-ended, focused or rigidly structured, allow the researcher to move beyond the original questions as the interview develops and allow for probing interviewees for clarification. Also, the interviewee has an opportunity to seek clarification of the intent of a particular question.

Data from interviews can be content analyzed to determine the nature and extent of adherence to the curriculum by a particular teacher. This approach is particularly well suited to studies of mutual adaptation which recognizes that teachers will adapt new approaches, resources and curriculum materials to suit their situation.

The questionnaire in contrast is limited by the questions which the researcher must set in advance. Furthermore, the researcher does not know if the questions or terminology used in the questions has the same meaning for the respondents as for the researcher.

Fullan and Pomfret suggest that both document analysis and the focused interview are promising techniques in that they allow a more thorough assessment than questionnaire surveys but are not as time consuming as observation methods.

Fullan and Pomfret drew three general observations in relation to measurement techniques from their review of fifteen implementation studies. It is probably desirable to employ more than one measurement technique in any research study, given that each has advantages and disadvantages. Different methods may well uncover different information which provides the researcher with the opportunity for cross validation. Their second observation suggests that some dimensions of implementation can be directly observed (eg. use of materials, changes in structure and in behavior), while others can only be inferred (eg. knowledge and understanding, philosophy) or determined through questioning and examination of documents. Third, there is no intrinsic reason why any of the methods cannot be used with either true fidelity or the mutual adaptation approach.

Curriculum implementation studies that use strictly analytical methodologies have the disadvantage of not being able to reveal why some aspects of an innovation are adopted while others are ignored. This weakness lies in the nature of empirical studies in that the data are reduced to average ratings and percentage scores. Consequently, the researcher may be left with a limited understanding of a particular classroom situation. Aoki (1984) supports this view in that he contends that the major weakness of analytical studies (which tend to be one-dimensional) is that the uniqueness and messiness of any lived situation tends to be "reduced out". This suggests that more than one measurement technique is necessary to gain an understanding of the interpretive process used by mathematics teachers in translating the intended curriculum to students.

Shavelson and Stern (1981) conducted a review which assessed the progress made in research on teachers' pedagogical thoughts, judgments, decisions and behavior during the nineteen seventies and they identified areas of substantive and methodological research needed to improve the practice of teaching. They identified a number of methodologies, of which policy-capturing and stimulated recall are of some interest to the current study. In a policy-capturing study, teachers are given descriptions of hypothetical

situations. These hypothetical situations vary with regard to a number of factors, which are systematically manipulated from one description to another.

Teachers' judgments would then be regressed on the students' characteristics and the resulting multiple-regression equation would be interpreted as a model of the teachers' decision-making process. This model would allow the relative weights of various factors in the teachers' strategy for decision-making to be assessed. Shavelson and Stern point out that there are several problems in the application of this approach. One, there is a problem with the generalizability of this model to more realistic situations because studies of this kind are typically carried out in laboratories. Furthermore, teacher responses to hypothetical situations may not be an accurate indicator of how they would respond in real life situations. (Deutscher, 1966) Second, regression analyses typically aggregate data from all of the teachers in a study. However, in doing so with ordinary least squares regression programs, the tacit assumption is made that each teacher has exactly the same policy. Associated with this is the need for a large sample in order to have enough responses for each of the combinations of factors if the data analysis is to be meaningful.

Stimulated recall is typically used when the process-tracing technique would interfere with the subject's performance on a task. The researcher either audiotapes or videotapes the lesson, which is then played back to the teacher by the researcher. The teacher is assisted in recalling the covert mental activities that accompanied the overt behavior as expressed through the teaching activities in the lesson. When stimulated recall is not possible, the researcher may use the points noted during the classroom observations in stimulating teacher reflection during the subsequent interviews.

3. Research on Teacher Decision-Making

A. Curriculum Implementation

Implementation as an important phenomenon in its own right has only received attention in the last decade or so. Fullan and Pomfret (1977) indicate that prior to that "...commentary and research on planned school change revolved almost exclusively around specifying the properties of the innovation, assessing outcomes, and to a lesser extent, planning changes and getting people to agree to try them." (p. 336)

What exactly is implementation and why is it so important in the educational change process? Fullan and Pomfret (1977) define implementation as referring to the actual use of an innovation or what an innovation consists of in practice. (p.336) They see this as being different from both the intended or planned use and from the decision to use, the latter being defined as adoption.

In terms of its importance, Fullan and Pomfret (1977) state four reasons: (1) We simply do not know what has changed unless we attempt to conceptualize and measure it directly. The point being that it is erroneous to assume that the move between the planning stage and the classroom has been unproblematic. Neither can one assume that the innovation has been implemented as intended or planned. (2) We need to examine implementation in order to better understand the reasons why so many educational changes fail. This would permit identification of some of the most problematic aspects of bringing about change. (3) Failure to study implementation may result in implementation being ignored, or else being confused with other aspects of the change process such as adoption, or even the confusing of the determinants of implementation with implementation itself. (4) Unless implementation is examined separately, it may be difficult to interpret learning outcomes and to relate these to possible determinants.

In his book, The Meaning of Educational Change, Fullan (1982)

maintains that:

One of the great mistakes in North America in the late 1960s and 1970s was the naive assumption that involving some teachers on curriculum committees or in program development would facilitate implementation, because it would increase acceptance by other teachers. ...As far as most teachers were concerned, when the change was produced by fellow teachers it was just as much **externally experienced** as if it had come from the university or government. In fact, it could be more aggravating if teachers who had developed the change were seen as getting special rewards and recognition, or if the teacher developers saw themselves as innovative and their colleagues as somewhat resistant or slow to catch on to the great new ideas they had produced." (p. 113, emphasis in the original)

Recognizing that implementation refers to the actual use of an innovation in practice, the above comment should not be surprising. The researcher's own experiences support Fullan's contention. How each teacher copes with an innovation in practice is a highly personal experience and teachers must have the opportunity to work through this experience in a way that is most meaningful to them. In actual practice this means that teachers need two to five years before they implement the innovation in practice. A change in practice for the teacher invariably means at least three things: (1) the possible use of new or revised materials, (2) the possible use of new teaching approaches and (3) the possible alteration of beliefs. (Fullan, 1982; p. 30)

Fullan's (1982) examination of a number of curriculum implementation studies revealed that teachers are using numerous criteria in assessing any change or proposed innovation. These are: (1) Does the change potentially address a need? Will the students be interested? Will they learn? (2) How clear is the change in terms of what the teacher will have to do? (3) How will it affect the teacher personally in terms of time, energy, new skill, sense of excitement and competence, and interference with existing priorities?

The success of an implementation is largely determined by the extent to which teachers care to and are allowed to interact. Support for this statement is provided by a number of researchers (Berman & McLaughlin, 1978; Hall &

Loucks, 1977; Fullan, 1982; Werner, 1980) and it appears that research on implementation has demonstrated beyond a shadow of a doubt that these processes of sustained interaction and staff development are crucial regardless of what the change is concerned with.

In the preface to his book, Fullan (1982) argues that the central issue in educational change is not the number of new programs developed, but rather the problem of finding meaning in change. If change attempts are to be successful, individuals and groups must find meaning concerning what should change as well as how to go about it.

Fullan (1982) maintains that the lack of shared meaning about an innovation is the chief cause of failure of implementation.

The main reason for failure is simple - the developers went through a process of acquiring **their** meaning of the new curriculum. And once it was presented to teachers, there was no provision for allowing them to work out the meaning for themselves of the changes before them. Innovations which have been succeeding have been doing so because they combine good ideas with good implementation support systems. (p. 103, emphasis in the original)

How teachers' interpret the Math 30 curriculum will largely depend upon the meaning that this curriculum has for them. Implicit in this is, of course, the assertion that should the interpretation differ from the one intended, the curriculum developers should make every attempt to examine its causes. What has been done by way of enhancing shared meaning should be the central focus of such inquiry.

B. Curriculum Decision-Making

Interpreting the Math 30 curriculum to students is equivalent to making curriculum decisions. A number of studies (Leithwood, Ross & Montgomery, 1982; Wahlstrom, Regan & Jones, 1982 and MacDonald & Leithwood, 1982) have identified factors having greatest influence on teachers' curriculum

decisions as emanating from within the classroom.

Leithwood, Ross & Montgomery (1982) reviewed the studies on teachers' curriculum decision-making and concluded that the emphasis in all of them had been in two areas: (1) identifying influences which shape the teachers' decision-making process and its outcomes and (2) determining the range of issues considered by teachers in their decision-making.

Research dealing with teacher decision-making suggests at this point that:

1. the number of influences on teachers' decisions will be small and the major ones will include the teacher's own experience, the opinion and behavior of students, and, to a lesser degree, the principal and other teachers in the school.
2. the influence of teachers' school colleagues on classroom decisions will vary widely depending in part on task requirements and norms for collegial interaction. (p. 14)

Research related to the range of issues spanned by teachers' curriculum decision-making has focused on the autonomy of the classroom teacher in areas of instruction and classroom management. The research findings reported by Leithwood, Ross & Montgomery are as follows:

1. The individual teacher is virtually autonomous in determining:
 - a. the nature of teacher-student (and to a lesser extent student-student) classroom interaction;
 - b. the kinds of classroom activities students will engage in to achieve curriculum outcomes;
 - c. the pace, timing, and total time allocation of instruction for a given curriculum area;
 - d. the frequency and timing of testing.
2. Teachers share decision-making responsibility with others in the areas of:
 - a. objectives;
 - b. choice of curriculum material;
 - c. choice of teaching strategies.

3. Principals and staff groups dominate classroom decisions about:
 - a. global concepts and the basic outline of the curriculum, and
 - b. the criteria to be used in assessing the curriculum and the students.
 (p. 15)

In addition to reviewing the research on teachers' curriculum decision-making to date, Leithwood, Ross & Montgomery (1982) report the results of their own study which intend to test the present knowledge on teachers' curriculum decision making and attempt to refine that knowledge.

The results of their study largely support the findings from earlier research with some noted differences. The factors perceived by teachers to be the strongest influence on the curriculum decisions are:

1. Your past experience (What you have found "works").
2. Awareness that students will experience satisfaction or pleasure, or their special needs or interests will be met, with certain alternatives.
3. Your attitude, special interests, philosophies.
4. Students.

Another nineteen factors of medium influence and seven factors of weak influence on teachers' curriculum decision-making are identified in this study.

The noted difference is that Leithwood et al found the principal exercised little direct influence on teachers in curriculum decision-making, whereas earlier findings attribute greater influence to the principal. Furthermore, Leithwood et al can not support the influence of colleagues in relation to teaching methods and assessment tools, but agree with their influence in relation to objectives. With respect to the range of issues included in teachers' curriculum decision-making, the study by Leithwood et al confirmed the earlier findings about the autonomy of the classroom teacher.

The study conducted by Wahlstrom, Regan & Jones (1982) attempts to determine the relative influence of teacher beliefs on classroom teachers' practice in assessing student achievement in English, mathematics, science,

history and geography at the junior high school level. They suggest that as behavior becomes less socially visible the effect of social support on teachers' classroom practice will decrease. This implies that factors in close proximity to the teacher are relatively strong in their potential influence on teacher decision-making. In their review of the research, they found evidence to suggest that teachers modify innovative curricula to bring them in line with their own system of belief. This clearly suggests that the success of attempts to improve practice, whether initiated from within the classroom or by external agents, depends to a significant degree on their compatibility with teacher beliefs.

Thompson (1983) conducted case studies involving three junior high school teachers. Her findings suggest that there is strong reason to believe that in mathematics, teachers' conceptions (their beliefs, views, and preferences) about the subject matter and its teaching play an important role in affecting their effectiveness as the primary mediators between the subject and the learners. She concludes by saying that:

Teachers possess conceptions about teaching that are general and not specific to the teaching of mathematics. They also have conceptions about their students and the social and emotional make-up of their class. These conceptions appear to play a significant role in affecting instructional decisions and behavior. For some teachers, these conceptions are likely to take precedence over other views and beliefs specific to the teaching of mathematics. (p. 125)

In the study conducted by MacDonald and Leithwood (1982), the thrust was toward an explanation of influences on teachers' curriculum decisions and to relate these reasons to a plausible pattern of teacher needs. The research was focused on the following questions:

1. What basic needs do teachers attempt to fulfil through their classroom curriculum work?
2. How do teachers go about trying to fulfil these needs?
3. What aspects of their professional world do teachers perceive to be most helpful in fulfilling these needs?

While the answers to these questions are not entirely unambiguous, they suggest, however, an interdependent pattern of relationships among needs. At the core of this pattern is an overriding need for achievement, defined in terms of assisting students to become more cognitively competent. Teachers recognize that many student characteristics are outside their control which influence the extent and nature of competence possible. Therefore, the criteria for success vary on the basis of perceived student potential.

MacDonald and Leithwood maintain that with achievement as the core need and student cognitive growth as the sole criterion of success, teachers require reliable sources of information about the effectiveness of their classroom behavior in meeting the criterion. Information is required on a short-term basis so that classroom behavior can be continuously adjusted to maximize the probability of success.

They pointed out that many teachers use student interest and the quality of student responses as criteria in evaluating their own methods and content selection. Since spontaneous student responses are considered the most direct and sensitive form of feedback about the extent of achievement, teachers attach considerable weight to the fulfillment of affiliation needs with the students.

MacDonald and Leithwood found that at the curriculum planning stage, trusted peer colleagues may be of value in increasing the chances of teacher success by providing reactions to a teacher's instruction where they have observed it or by indicating the probable value of proposed teaching strategies on the basis of their own direct experience. Such feedback depends on a good working relationship with both students and colleagues.

Standardized tests are viewed as sometimes useful sources of confirmation about student progress, contributing to the fulfillment of teacher achievement needs. According to MacDonald and Leithwood, an identified influence on curriculum decision-making can be viewed as either an instrument for the fulfillment of achievement need or as an indicator about the extent to which the achievement need is being fulfilled. Teachers consider those

influences that serve neither purpose as low-ranked influences.

4. Research on Mathematics Curricula In Practice

Mathematics curricula in practice are an important source in investigating the nature of mathematics being taught, the manner in which the curriculum is being translated to students and the effects of that curriculum or teaching on students.

A. British Columbia Mathematics Assessment

In the spring of 1977, over one hundred thousand students from grades 4, 8 and 12 wrote the tests prepared for the 1977 British Columbia Mathematics Assessment. (Robitaille & Sherrill, 1977) In addition, almost three thousand teachers of mathematics at seven different grade levels (1, 3, 4, 7, 8, 10, 12) completed comprehensive questionnaires dealing with numerous aspects of the teaching and learning of mathematics.

The mathematics assessment served to inform the public of the strength and weaknesses of the public schools in that province, to assist curriculum developers in improving curricula and developing resource materials, and to provide researchers with a source of researchable questions concerning the teaching and learning of mathematics. The summary of the findings will focus on the grade 12 students only and those aspects relevant to the current study.

At the grade 12 level, the student population was divided into three subpopulations for a number of the analyses because of differences in mathematical background among the students. The three groups were: (1) those students who had their twelfth year of mathematics, (2) those students who had their eleventh year of mathematics and (3) those students who had their tenth year of mathematics.

The mathematics assessment consisted of four major components: the

Goal Assessment, the Student Tests, the Interpretive Analysis and the survey of Instructional Practices. The survey of instructional practices is of particular interest. The four major sections included in this survey deal with: classroom organization, use of textbooks, classroom instruction and the importance of certain mathematical content objectives. With regard to classroom organization, the results indicate that a considerable amount of time is spent in preparing for and teaching mathematics classes. Teaching of mathematics is highly traditional in character. The most frequently used teaching techniques are total class instruction and teacher explanation. Regarding classroom activities, the most prevalent at the secondary level are individual work, textbook exercises and teacher explanation. The use of activity centers and creative projects for the teaching of mathematics is very limited at secondary levels. The results indicate that few organizational innovations are being used in the mathematics classes of British Columbia.

The findings clearly indicate that the use of textbooks in mathematics classes is virtually universal at all grade levels. On the whole, teachers are quite satisfied with the textbooks they are using, although many of the textbooks are no longer on the prescribed list. All teachers use their mathematics texts primarily as sources of exercises and expressed that they particularly want textbooks to provide materials for drill and practice. Teachers' rating of prescribed texts with regard to provisions for problem-solving and computation seemed to be a serious concern. It appears that teachers do not require their students to read very extensively from their mathematics textbook.

The results with regard to classroom instruction are rather revealing. Teacher responses to five content areas indicate that secondary mathematics teachers spend the most time on problem-solving and algebraic concepts and the least time on metric measurement. Rating the usefulness of a number of teaching resources, all teachers agreed that the students' textbook was a useful resource. Teachers were asked to rank sixteen factors purported to affect mathematics instruction. Secondary mathematics teachers gave high priority to the following four factors: reduction of class size, textbooks more suited to

instructional needs, ability grouping of students for classes and reduction of total pupil load. The data gathered concerning the frequency of use of selected media, materials and methods in the teaching of mathematics indicate that the chalkboard is the most popular teaching aid among teachers of secondary mathematics. In terms of frequency of use of methods for presenting mathematics instruction, total class instruction and individualized instruction ranked first and second, respectively. The use of computers occurs in only a few mathematics classes at the secondary level. The importance attached to eight evaluation techniques by teachers, revealed that teacher-prepared tests were far above any other evaluation technique. They also attached considerable importance to evaluating of performance on assignments and teacher observation of students' work. Secondary teachers do little designing of mathematics programs in their school, but rather work with existing programs. A substantial number of secondary teachers felt that they did not have access to Learning Assistance Centers for mathematics.

With regard to the importance of selected curriculum objectives, the grade 12 teachers did not receive lists of possible mathematics learning outcomes for their specified grades. However, they participated on the question of minimal mathematics objectives for graduation from secondary school and the results are as follows:

1. teachers put a high premium on graduates being able to perform the four basic operations with whole numbers, fractions and decimals.
2. teachers felt it was essential that graduates be able to apply their mathematical knowledge in both physical-world and consumer-related situations.
3. teachers felt it was essential for graduates to be able to use the metric units of measurement.

B. National Science Foundation Studies

A research team headed by Stake and Easley (1978) conducted case studies that focused on the teaching and learning of science, mathematics and social science education in American public schools during the 1976-77 school year. The research approach used by Stake and Easley is of some interest in this current study in that it skillfully employed qualitative and quantitative research methodologies to obtain a complete picture of classroom practices.

Eleven high schools and their feeder schools were selected, covering a wide variety of geographic, demographic, social and economical situations. To confirm findings of the ethnographic case studies and to add special information, a national stratified random-sample of about 4000 teachers, principals, curriculum supervisors, superintendents, parents and senior class students were surveyed. Survey questions were based on observations at the eleven case study sites. Field researchers spent anywhere from four to fifteen weeks on site, gathering data. They were instructed to find out what was happening, what was important in science, mathematics and social science.

The findings of this study suggest that each school was different in important ways and that each teacher made unique contributions. Overall, it was felt that there was a yielding to an increasing emphasis on basic skills (eg. computation). Only courses for the college-bound students seem to resist this pressure. Students in mathematics seem to be very conscious of their grade as is reflected by the statement of one teacher, "...a lot of students are strictly looking at their achievement in terms of a grade." (p. 4-28) While some teachers thought they were teaching the structures of mathematics, several of the students expressed that they were confused, found it hard or lacked interest. Teachers rarely used the inquiry approach and mathematics was presented in a way that promoted the image of the teacher being competent. This also ensured a maintenance of student respect and required students to work hard. School people and parents were generally supportive of what was chosen to be taught, complaining occasionally that it was not taught well enough.

The textbook was usually seen as the authority on knowledge and the guide to learning. Teachers were perceived to be the authority on both social and academic decorum. While there was relative freedom to depart from district syllabus or community expectations, teachers seldom exercised either freedom.

An important idea that emerged from the Stake and Easley case studies was the hypothesis that what teachers do with subject matter is determined by how it sustains and protects them in the social system of the school.

C. Second International Mathematics Study

Canada, being represented by British Columbia and Ontario, participated during the 1980-81 school year in the Second International Mathematics Study (SIMS). The results reported in this section will deal with British Columbia only. (Robitaille, O'Shea & Dirks, 1982) SIMS is a project of the International Association for the Evaluation of Educational Achievement (IEA). The specific purpose of SIMS is three-fold:

1. to investigate the nature of mathematics being taught in those countries participating in the study,
2. to compare the ways in which mathematics was taught and
3. to study the effects of that teaching on students' learning of and attitude toward mathematics.

The international study is a broadly-based, comparative investigation of the mathematics curriculum as prescribed (intended), as taught (implemented) and as learned (attained). All three dimensions of the mathematics curriculum were investigated. An analysis of the intended curriculum was conducted by examining curriculum guides and related documents from each participating country. In addition, several lengthy questionnaires were completed in each country and all of this information was utilized to construct a composite view of the intended curriculum.



For the study of the implemented curriculum, a number of questionnaires for teachers were developed. These included the Classroom Process questionnaires, a series of unique instruments designed to collect highly specific information from teachers regarding the methods used in teaching specific topics in the curriculum. These Classroom Process questionnaires (CPQ) have been used to produce a description of mathematics teaching in British Columbia at the secondary level. In order to assess student outcomes, test items, questionnaires and attitude scales were developed. Unfortunately, the results from the CPQ had not been linked to student achievement.

While each country has its own set of reasons for participating, British Columbia focused its interest on curriculum analysis and teaching process aspects of the study. Two populations of students were identified. Population A was defined to consist of all those students enrolled in grade 8 and Population B consisted of all those grade 12 students who were studying college-preparatory mathematics in grade 12. Only Population B will be discussed in this section. Following the examination of all curriculum guides from participating countries, the composite view of the intended curriculum included trigonometry, analytic geometry, algebra and calculus. In British Columbia, the calculus questionnaire was not used because calculus is not part of the core curriculum.

Due to the complexity of the study and the wealth of information that has been generated by it, it is only possible to describe some of the major findings related to the curriculum as taught or implemented. They are:

1. There are very substantial differences in the amounts of time teachers spend on the different components of the curriculum. (eg. in trigonometry the range was from 10 to 56 periods, with the mean being 29)
2. Approximately 40% of the mathematics teachers felt that trigonometry should be a separate semester-long course. Only 3% disagreed.
3. Algebra 12 teachers spend substantial amounts of time reviewing material that was covered in earlier courses.

4. Relatively little time appears to be spent on developing students' ability to construct graphs of functions.
5. Almost all Algebra 12 teachers provided some form of proof or informal justification in presenting formulas or identities, but they only expected to recall the formulas and apply them.
6. Nearly all Algebra 12 teachers (97%) permitted calculators in class and some (16%) even used microcomputers.
7. Algebra 12 teachers adhere closely to the course outline as contained in the curriculum.
8. Algebra 12 teachers are more likely to use a particular teaching approach because they feel it will be easier for students to understand than because they think students will enjoy it.
9. In a typical, Algebra 12 class, 25% of the time is spent on correcting homework, 40% on lecturing and 20% on individual seat work by students. Little time is allocated for small group work.
10. Algebra 12 teachers typically do not differentiate between levels of ability in their teaching. Most teachers teach the same content in the same way and give the same assignment to all students. (Robitaille, O'Shea & Dirks, 1982, p. 162-164)

The results of this Second International Mathematics Study suggest that there is a wide gap between the classroom practices and the intentions of curriculum developers.

III. DESIGN OF THE STUDY

In the previous chapter it was stated that a research design is the logic that links the data to be collected to the initial questions of a study. This chapter outlines the case study design used in this study with the focus on six broad components: (1) the study's questions, (2) the underlying propositions or purposes of the questions, (3) its units of analysis, (4) research instruments and data collection, (5) the logic linking the data to the propositions, and (6) the criteria for interpreting the findings.

A. The Study's Questions

Case studies are the preferred strategy when "how", "what" and "why" questions are being posed. (Yin, 1989) The current study poses five major research questions all of which are of the "how" and "what" type.

1. What interpretation do teachers give of the intended curriculum as represented by the trigonometry unit of the Mathematics 30 curriculum?

This question has two dimensions. The first dimension is exploratory in nature and the goal is to develop propositions for further inquiry. The exploratory nature of the question requires conducting a systematic search through or into the teachers' interpretation of the curriculum. It is essentially investigative in nature. The second dimension of this question is one of prevalence. It is of the form of a "how many" or "how much" line of inquiry. The first dimension is examined using classroom observations and interviews. The second dimension of the question is answered through a variety of survey strategies. This question is examined by contrasting the teacher interpreted curriculum against the intended curriculum. The interpreted curriculum is the focus of this study and it describes the classroom processes, actions and activities of a teacher which result from that teacher giving meaning to the

intended curriculum. The intended curriculum is the goals, statements and learning objectives as well as the philosophy and rationale as outlined in the Mathematics 10-20-30 Curriculum Guide (1983).

2. What are the classroom practices of teachers and to what extent are these practices successful?

This question is explanatory in nature. It deals with the operational links within the current study as well as frequencies or number of incidents. This question examines the various facets of the causal arguments. (Yin, 1984) Multiple sources of evidence are used to answer this question. This question focuses on the actual classroom practices and processes and it relates these to student outcomes.

3. What are some of the factors which influence the teachers' interpretation of the trigonometry unit to students?

Although this question is exploratory in nature, it also conceals a question of the "why" line of inquiry, thus requiring an explanation-building procedure. In order to explain some of the phenomena identified, a set of causal links about the phenomena are stipulated. The data obtained from classroom observations, interviews and survey strategies are used as sources of evidence in the identification of factors that influence the interpretive process used by classroom teachers.

4. What is the relationship between the various factors identified in the study that affect the teachers' interpretation of curriculum materials?

The use of multiple sources of evidence provide the basis for answering this question. Its purpose is to examine the causal relationship between a given factor and the resulting interpretation of curriculum materials by teachers and to develop propositions for further inquiry.

5. What is the nature of teachers' functional paradigm as characterized by their interpretation of the Mathematics 30 curriculum?

This is the central research question and it involves investigating the role of mathematics teachers' functional paradigm in translating curriculum materials into classroom practice. Furthermore, it seeks to identify some of the factors which influence the teachers' interpretation of curriculum materials. An examination of each individual case and the multiple case report form the basis for answering this exploratory question. In addition, propositions are developed for further inquiry.

B. The Underlying Propositions or Purposes of the Questions

This second component of the research design is of considerable importance in that it directs attention to those things to be examined within the scope of this study. The current study began with an interest in the teachers' interpretation of the Math 30 curriculum. From this general interest five broad research questions evolved which were of the "how", "what" and "why" nature. While the nature of the questions led to the case study as the appropriate research strategy for the current study, the questions did not point to specific areas or aspects that needed to be studied. It was through statements of propositions or specific purposes which were closely linked to the five broad research questions, that specific areas or aspects of study were identified. These propositions or specific purposes also assisted in the design of the research instruments.

Yin (1984) stated that some studies may have a legitimate reason for not having any propositions. This is particularly the case in which a topic is the subject of exploration. However, he points out that every exploration should still have a purpose which should be clearly stated as well as the criteria by which an exploration will be judged successful. The research questions raised in the current study vary in nature and therefore some are linked to specific propositions and others are accompanied by a stated purpose or both.

Linking Research Questions to Propositions and Purposes

In this section each of the five major research questions is stated and followed by a statement of purpose and propositions where applicable.

Research Question 1: What interpretation do teachers give of the intended curriculum as represented by the trigonometry unit of the Mathematics 30 curriculum?

This question is exploratory in nature and it seeks to reveal the nature of the teachers' interpretation of the intended curriculum. Three different types of data gathering approaches are used to answer this question. The purpose of this question is to examine how closely the teacher-interpreted curriculum adheres to the intended curriculum. In terms of a propositional statement, the current study anticipates that the teachers' interpretation of the intended curriculum is influenced by their desire to prepare students well for the Mathematics Diploma Examinations.

Research Question 2: What are the classroom practices of teachers and to what extent are these practices successful?

This question is explanatory in nature, dealing with the operational links within the current study as well as mere frequencies or incidents. Four different data gathering approaches are used to answer this question. Some of the instruments were used to examine the operational links between teacher-interpreted curriculum and the attained curriculum.

Within the scope of this question several propositions are stated below which draw attention to a number of issues and areas that are relevant to this question. These propositions are not just helpful with respect to achieving the right focus, but they also assist in developing research instruments, in gathering relevant research evidence and in avoiding to collect too much. This research question is guided by the following propositions:

- a. Most trigonometric formulas are derived without using the formal proof and students are expected to merely apply a formula when given.
- b. Teaching strategies and student evaluation/marking schemes used by teachers are selected with pedagogical efficiency in mind.
- c. The use of resource materials in the mathematics classroom is confined to the prescribed textual resources and recommended reference.
- d. The allocation of time to various topics or activities in trigonometry varies depending on the stated or perceived importance of each.
- e. The time allotted for the elective component of the mathematics curriculum is used for the purpose of teaching and reviewing materials from the core component.
- f. The nature of the review focuses only on materials from the core component and it emphasizes the preparation of students for the Mathematics Diploma Examinations.

Research Question 3: What are some of the factors which influence the teachers' interpretation of the trigonometry unit to students?

Although this question is exploratory in nature, it also conceals a "why" question, thus making it explanatory. In order for a particular agent to qualify as a factor, it must have a causal relationship with a particular interpretation. Conversely, one could ask "why does a teacher have a particular interpretation?" and then trace this interpretation to a particular factor. However, this question is intended to be about prevalence, as in "How many factors are there that influence the teachers' interpretation of the trigonometry unit to students?", hence a survey strategy is used. Two distinct data gathering approaches are used to answer this question. The purpose of this "exploration" is to examine broad categories of background information on teachers, their teaching situation, their curriculum decision-making, their choices of curriculum materials and students which serve as the foundation to examine the relationship between the factors identified and the effect they have on the

teachers' interpretation of curriculum materials. This question is examined at the individual case level and the provincial sample level.

Research Question 4: What is the relationship between the various factors identified in the study that affect the teachers' interpretation of curriculum materials?

This research question is also exploratory in nature and it is examined across multiple cases and in relation to the provincial sample. Case level and sample information is used to identify the relationship between various factors that affect the teachers' interpretation of curriculum materials.

Research Question 5: What is the nature of teachers' functional paradigm as characterized by their interpretation of the Mathematics 30 curriculum?

To answer this exploratory question, data are extracted from the multiple case summaries and backed by research literature. The purpose of this question is to examine the nature of the teachers' functional paradigm or techniques, exemplars and routines that are shared by a group of mathematics teachers and that guide their classroom practice. The focus in the current study is on commonalities across teachers rather than differences.

C. Unit of Analysis

The unit of analysis, or the case, is related to the way the initial research questions are defined. Determining the unit of analysis is fundamental in defining the "case". The "case" in this study is defined to be the individual classroom teacher. In all, three individual cases from the same school system are studied, making it a multiple-case study using a replication approach. While the individual classroom teacher is the unit of analysis, each case also includes embedded units of analysis. These include the student results from an attitude test and the diploma examination for each class. The results for each

class are not pooled across the cases but they are part of the findings or information base for each individual case.

Another unit of analysis is the sample of six teachers from across the province who responded to a survey. This unit of analysis is neither embedded in nor a subunit of the cases, but it stands by itself for the purpose of placing the three cases within the context of other practicing teachers.

D. Research Instruments and Data Collection

This section describes the research instruments in relation to the study's questions and it deals with the data collection procedures related to (1) case study teachers, (2) classroom students, and (3) the provincial teacher sample.

Research Instruments in Relation to the Study's Questions

In this section the development of the classroom observation sheets, the teacher interview questions, the Classroom Practices of Trigonometry Teachers Survey (CPTT), the Attitude Toward Mathematics Test (ATMT) and the Mathematics Diploma Examination (MDE) is described.

Classroom Observation Sheets: Three observation sheets were prepared in advance and used for each classroom observation. These observation sheets served three purposes: (1) to provide a focus for the classroom observations, (2) to facilitate note taking and (3) to facilitate data analysis. (Schatzman & Strauss, 1973; Yin, 1984) One of the observation sheets deals with the physical environment (Section 1) which relates to the structure and organization of the mathematics classroom, the second sheet (Section 2) relates to the teaching strategies used during the lesson and the third sheet (Section 3) relates to the degree to which observable problem solving indicators are present in the classroom. The outline of the observation sheets is

used to organize the field notes. A copy of the instruments is contained in Appendix A.

Section 1, the physical environment, focuses on all the features of the room where the observed classes are taught. More specifically, the first point calls for a description of the main features of the room, followed by points regarding windows, blackboards, tackboards, audiovisual screen and equipment, mathematical equipment, shelves and desks. Adequate room is also provided to describe and comment on mathematical displays, posters, mathematical resources and other features.

Section 2 deals with teaching strategies, focusing on teacher and student activities and the use of curriculum resources during the mathematics lesson observed. In addition, ample space is left for noting teacher comments relative to their perceptions of mathematics, students and teaching. The general comments include observation points about the teacher's use of mathematics materials, problem solving, students or other noteworthy points. The final point lists the situational interview questions that are generated during the observation, and that are to be discussed with the teachers during the subsequent situational interview.

Section 3 contains a problem-solving scale which uses a five-point rating scale for each of the four problem-solving indicators, based on observable teacher behavior. The indicators are Understanding the Problem, Devising a Plan, Carrying out the Plan and Looking Back. The degree to which the teacher exhibits any or all of the four problem-solving indicators is recorded for each observation. The observable teacher behaviors that qualify as evidence for each of the indicators are listed on the Classroom Observation Sheet. The classroom observations, with their focus on the physical environment, teaching strategies and problem solving, are an important source of evidence in answering the research questions.

Teacher Interview (TI) Questions: The questions were developed for the purpose of conducting a focused interview with each of the three teachers

observed. These questions focus the discussions on certain themes. They provided a common reference point and they frame the information about the mathematics program that is collected from each teacher. Although the interview is structured and focuses on thirteen questions, the interviewees are in no way limited in the depth or breadth of their response. The questions are designed so that classroom observation could be compared to provide a triangulation type of validity check. (Schatzman & Strauss, 1973) Throughout the interview, a sense of collegiality and trust is to be preserved which is essential to establish research conditions which are favorable for validity. (Schatzman & Strauss, 1973) Every attempt is made not to reflect evaluative judgements on the teachers' classroom strategies, but rather a message is to be communicated that their opinions are respected and worthy of consideration. The following outline is used to introduce and guide the teacher interviews:

My interest is in examining the interpretive process used by Math 30 teachers in translating the Math 30 curriculum to students. How the teachers interpret the Math 30 curriculum and why they interpret the curriculum in particular ways is of specific interest.

Thirteen interview questions are asked of each of the three teachers observed. The teacher interview questions were piloted, involving two classroom teachers, two mathematics consultants, one mathematics test development specialist, one specialist from the Student Evaluation Branch of Alberta Education and one curriculum specialist from Alberta Education. The pilot study was limited to the data collection instrument only and not to the procedures to be followed. Each pilot participant was provided with information regarding the nature of the study and the major research questions. Their responses to the interview questions indicated that they were suitable for the study, relevant, covering the domain of interest and nonthreatening to the teachers. This was certainly evident during the actual interview with the teachers observed. They responded with ease to each of the questions. Each interview was tape recorded and it provided an important source of evidence for

corroboration as well as understanding the interpretive process used by teachers in translating curriculum materials to students. In addition to recording the interview, note-taking occurred as well. This was considered important because it conveys to the interviewee that the researcher is interested in what was being said and it assists in the analysis of the transcripts.

The questions that were asked during each interview covered a number of general areas, including the nature of the content of the trigonometry unit, teaching strategies, resource materials, nature of student evaluation (eg. unit tests, quizzes, worksheets, homework), emphasis on problem solving and mathematical applications, nature of students, elective topics, factors affecting the teachers' interpretation, importance of various concepts and student difficulties.

Classroom Practices of Trigonometry Teachers Survey (CPTT): The CPTT was developed to gather additional information about teacher interpretation of curriculum materials from a larger sample of teachers than could be observed and interviewed in person. (Fullan & Pomfret, 1977) It was administered to six teachers selected from six geographic regions across the province and to each of the three teachers observed. This allowed the researcher to situate the three cases within the context of other practicing mathematics teachers.

Many of the ideas included in the CPTT were developed based on the research questions posed and the researcher's experience in classroom observations. Additional ideas for items were gleaned from questionnaires used in other education studies. (Robitaille & Sherrill, 1977; Stake & Easley, 1978; IAEEA, 1985) In all, the CPTT is comprised of ten parts and the items within each part are arranged in sequence which would appear logical to the respondents. A draft of the CPTT was prepared and piloted with seven people, two mathematics consultants, one mathematics test development specialist, one specialist from the Student Evaluation Branch and one curriculum specialist from Alberta Education. In addition, the CPTT was reviewed by one university

researcher experienced in survey research. Each of the pilot participants was familiar with the nature of the study and the major research questions posed. However, the pilot was limited to the instrument itself and not to the procedures to be followed during its administration. The researcher requested that the pilot participants focus on the completeness of each part and the whole instrument, on the appropriateness of each response category, on the time required for completion and on the clarity and readability of each question. In addition, they were asked as to whether there should be open-ended response categories.

The responses and comments of the pilot participants were used to guide revision of the instrument. One area that needed revision was the section on rating the adequacy of the teacher's background in a number of areas. The response category "excellent" followed by "good" was seen as too far apart. It was suggested that "very good" be chosen for "excellent". Other comments related to typographical errors and minor suggestions in clarity. In general, all pilot participants found the CPTT to be complete in all respects, appropriate and clear. A copy of the CPTT is included in Appendix B. The CPTT consists of ten parts with the nature and purpose of each part being as follows:

Part 1: Background Information. This part is designed to capture basic demographic information about mathematics teachers, the adequacy of the teachers' background in various areas, their perception of students' attitude toward mathematics and nature of their school and their community. All questions are written in multiple choice format.

Part 2: Trigonometric Subtopic Questionnaire (TSQ). This part deals with the mathematics content covered. The trigonometry unit is broken down into 32 subtopics and teachers are asked to respond to each subtopic according to three response codes. The first response code deals with the treatment of each subtopic in the teacher's class and provides five different response alternatives. The second response code assumes that the topic was taught or reviewed and from the nine response alternatives teachers are asked to indicate their first and second reason for teaching or reviewing the subtopic. The third response code assumes that the topic was neither taught nor

reviewed and from the nine response alternatives teachers are again asked to indicate their first and second reason for not teaching or reviewing the subtopic. The responses to this part of the CPTT provide the researcher with an indication of the teacher's interpretation of the curriculum in terms of the relevancy of subtopics and the reasons for teaching or not teaching the various subtopics.

Part 3: Trigonometry Objectives Emphasis Survey (TOES). The nineteen objectives listed in the trigonometry unit of the Math 30 Curriculum Guide (1983) are stated and followed by a five-point rating scale. Teachers are asked to indicate the degree of emphasis that each objective receives in their classroom.

Part 4: Trigonometric Formula Questionnaire (TFQ). The TFQ contains 17 items and it is designed to gain a deeper understanding of the teachers' interpretation of the trigonometry unit and their expectations of students for specific formulas in trigonometry. Three response codes with corresponding response alternatives are used. The first response code provides four response alternatives which deal with the justification of teachers relative to the presentation of formulas to the class. The second response code is based on the assumption that a formal proof is provided to the class and teachers are asked to indicate their primary reason for doing so by selecting from three response alternatives. The third response code provides four response alternatives, describing teacher's expectations of student mastery for each formula.

Part 5: Trigonometric Interpretation Questionnaire (TIQ). This questionnaire contains four interpretations of the trigonometric functions of sine, cosine, tangent, cosecant, secant, and cotangent. A brief description with an appropriate sketch is provided for each interpretation. The first response code provides three response alternatives, asking teachers to indicate the degree of emphasis given to each interpretation. The second and third response codes each have nine response alternatives and they require teachers to indicate the reasons for selecting and for not selecting these four interpretations, respectively. The information gleaned from the responses is to provide further

insight into the depth and breadth of the teachers' interpretation of the trigonometry unit.

Part 6: Skill Emphasis Survey (SES). This survey consists of ten skill statements, each is followed by a five-point response code signifying the degree of importance that teachers attach to each statement. The information obtained from this survey is to provide an indication of what interpretation teachers attach to the curriculum and the relationship between the interpreted and attained curriculum as expressed by the performance of students on the Mathematics Diploma Exam.

Part 7: Teaching Strategies Survey (TSS). This survey includes twenty-six different teaching strategies and teachers are asked to indicate how frequently they use each of these teaching methods in their mathematics classes. The items on the TSS are written in multiple-choice format with the following choices:

- A. Never or rarely
- B. About once or twice during a unit
- C. About once or twice during the week
- D. Almost every class period

The responses to the TSS provide an indication of the prevailing teaching strategies used by mathematics teachers (Bell, F. H. Teaching and Learning Mathematics, p. 222)

Part 8: Curriculum Material Selection Survey (CMSS). This survey focuses on the way teachers select and adapt curriculum materials to meet the needs of their particular class. Fifteen factors are listed and teachers are asked to identify the degree of importance that each factor has in selecting of or determining how to use curriculum materials in the classroom. The information gleaned from the CMSS is an important source of evidence to answer one of the major research questions dealing with the identification of factors that influence the teachers' interpretation of the trigonometry unit to students.

Part 9: Resource Material Survey (RMS). This survey tries to establish the frequency of use of various resource materials used by mathematics teachers. In all, eleven items are listed. Ten items identify specific resources and item eleven is open-ended, allowing teachers to list those resources that are not already listed. The three-point response scale ranges from "used frequently" to "not used at all". The intention of the RMS is to gather information on what materials teachers select to facilitate the teaching-learning process and on how they succeed in implementing their interpretation of the curriculum.

Part 10: Time Allocation Questionnaire (TAQ). This questionnaire consists of thirteen major topics or activities in trigonometry and teachers are asked to indicate how much time is spent in each of these. The intention of the TAQ is to determine how much instructional emphasis is attached to each of the major topics or activities by teachers.

Attitude Toward Mathematics Test (ATMT): The ATMT was adapted from the instruments used in the Second International Mathematics Study. It was administered to the students of each of the three teachers observed at the conclusion of the trigonometry unit. The test focuses on three general areas: (1) Mathematics in School, (2) Mathematics as a Process, and (3) Mathematics and the Student. Each area contains a number of statements followed by a five-point response scale. "Mathematics in School" includes a number of activity statements and students are asked to respond on two different dimensions. One dimension focuses on how students feel with respect to the importance of the activity and the other dimension seeks to determine the extent to which students like the activity. "Mathematics as a Process" includes a number of process statements and students are asked to express their level of agreement with each statement. "Mathematics and the Student" contains seventeen statements of personal feelings about mathematics. The purpose of administering the ATMT is to obtain an indication of how positive students feel about mathematics and the relationship between a class' attitude toward

mathematics and a particular interpretation of the curriculum. A copy of the ATMT is contained in Appendix C.

Mathematics Diploma Examination (MDE): The MDE is a province-wide examination based on the objectives of the Math 30 program. It is developed by the Student Evaluation Branch of Alberta Education. Math 30 is a course of 125 hours duration of which 26 hours is devoted to trigonometry. At the completion of the Math 30 course, students are required to write the mathematics diploma exam which includes topics on trigonometry; quadratic relations; logarithms; sequences, series, limits; statistics; and polynomial functions. Although students receive a composite mark for achievement on the MDE, the analysis of the results is done by topic, student, cohort, school system and province. This allows the researcher to examine the achievements of the three participating classes in trigonometry only. The MDE data for trigonometry are included for the purpose of examining the relationship between the teacher-interpreted curriculum and the attained curriculum. In addition, this data source is seen as adding to the "richness" of the case information.

Data Collection Procedures

Case Study Teachers: The researcher met with each of the three teachers individually and explained the research project. Aside from explaining the purpose of the study, it was pointed out that they were not obliged to participate. In addition, the researcher explained the expectations of each participating teacher. They were: (1) a willingness to be observed, (2) a willingness to be interviewed either during or immediately after each lesson observed, (3) a willingness to be available for an extensive interview following the teaching of the trigonometry unit, (4) a willingness to respond to the CPTT survey following the teaching of the trigonometry unit, and (5) a willingness to allow the administration of the ATMT to their students. Each teacher was advised that they had the option of withdrawing from this study at any time. Two teachers were visited 6 times for an eighty minute period over a five week time span

because their schools operated on a semestered basis. The third teacher was visited 10 times for a forty minute period over a ten week time span because his school operated on the ten month timetable

In all cases, the researcher was received as a fellow mathematics teacher and the teachers appeared completely at ease and comfortable. Prior to the start of this research project, the researcher visited all participating classes in order to increase the comfort level of the teachers and students.

During the teacher's presentation, the researcher sat in an empty desk at the back of the room taking notes and formulating questions to be asked. When students worked on assignments, the researcher moved around the room talking to students and responding to their questions. When convenient, the researcher also posed questions to the teacher during the class period. These situational interviews were an attempt to seek the quality of meaning teachers living in a situation give to their lived situation. (Aoki, 1984)

While the situational interviews took place during the class period or immediately after, the teacher interview which was an extended conversation was conducted during a prearranged time after school. All interviews were held in the teachers' classrooms. The teachers seemed quite at ease and responded openly and freely to the questions posed.

At the completion of the trigonometry unit teachers were given a copy of the CPTT survey with the request to complete it as soon as possible. All teachers returned it within one week following the completion of the unit.

The Classroom Students: The ATMT was administered to all the Math 30 students of the three teachers. The researcher explained the nature of the ATMT to each class prior to administering it. It was administered during a regular class period at a time convenient to the teacher, and it took ten to twenty minutes to complete. Student responses on the ATMT were tabulated and mean ratings, standard deviations and correlations, where applicable, were calculated for each item. At the completion of their Math 30 course, students were also required to write a provincially administered Mathematics Diploma Examination (MDE). The MDE is marked by Alberta Education and the results

are reported by topic. Calculations of mean ratings and standard deviations are done by topic, students, school, school system and province.

Provincial Teacher Sample: The sample of six Math 30 teachers was obtained by randomly selecting one high school from each of the six geographic zones in Alberta. These six geographic zones are coterminous with the six zones established by Alberta Education. A list of all high schools offering Math 30 was available by zone. The schools for each zone were listed in alphabetic order. A set of random numbers, ranging from 1 to the number of schools in each zone, was computer generated and the school corresponding to the first random number for each zone was selected. This selection method was chosen to ensure that no two schools would be located in the same school jurisdiction. A letter was written to the superintendent of each of the 6 jurisdictions, outlining the purpose of the study and requesting permission to contact the principals of the sample schools with regard to the CPTT. All superintendents confirmed their approval by telephone. In several cases the researcher was requested to phone the principal and the teacher involved to explain the nature of the study and the purpose of the CPTT. None of the six schools and teachers selected refused to complete the survey. The CPTT was returned by five schools within two weeks following the completion of the trigonometry unit. One school had to be phoned and the teacher indicated that it was nearly complete and it would be mailed within the next two days. It was received shortly thereafter. The inclusion of this provincial teacher sample was of considerable importance in this study because it allowed the researcher to situate the three cases within the context of a small sample of other mathematics teachers teaching the same curriculum.

Multiple-Case Design: The current study is a multiple-case design involving the individual case studies with embedded units and a random sample of six teachers. The elements of the design phase, data collection and analysis phase and the cross-case analysis phase are shown in FIGURE 3.

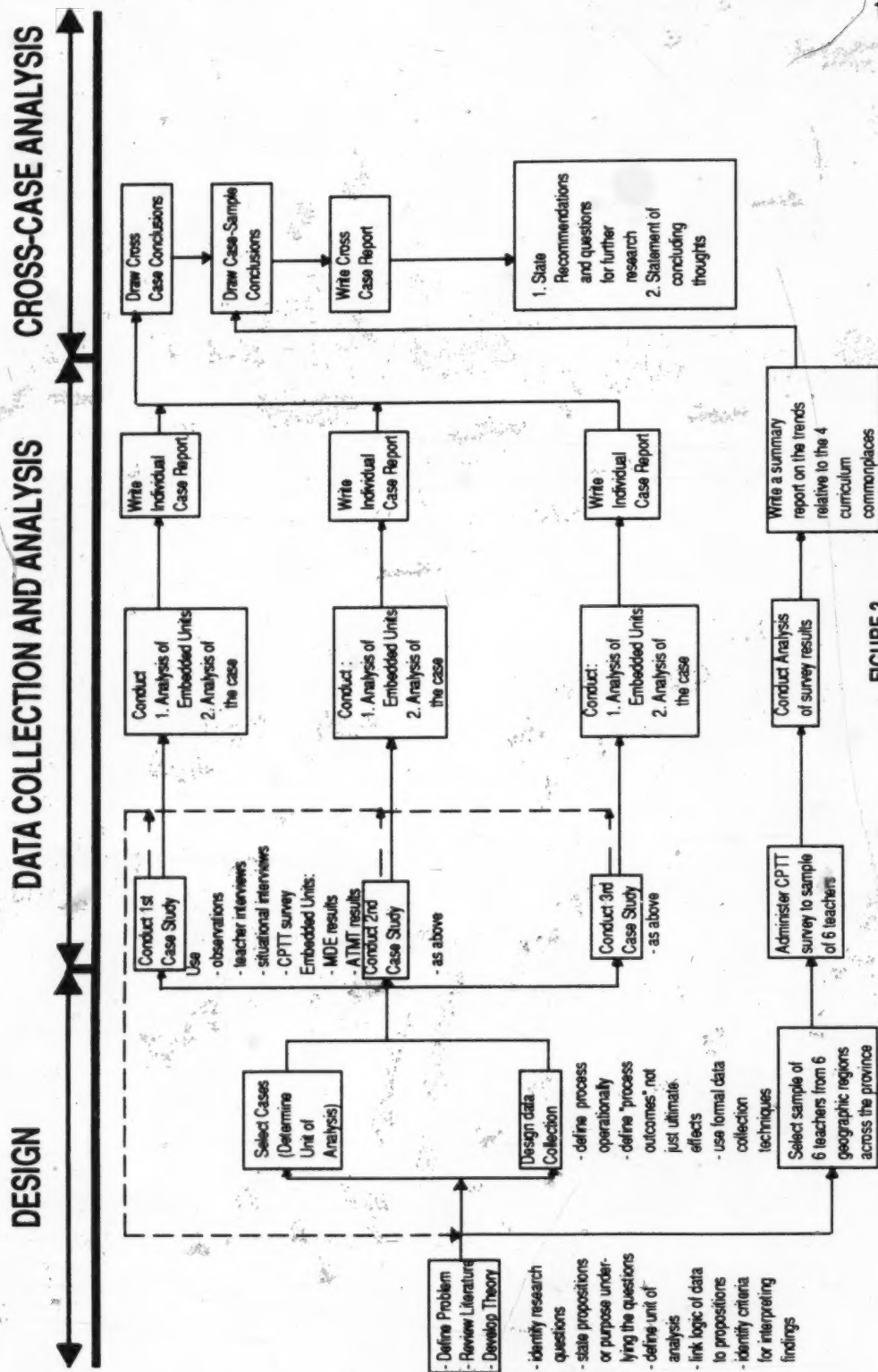


FIGURE 3
MULTIPLE-CASE DESIGN USING REPLICATION APPROACH

E. Linking Data to the Propositions

This section deals with the linking of the data with the proposition or major research questions. This is followed by a discussion of the data analysis and analytic strategies used.

Linking Data to the Questions

The overall objective of this study is to examine the interpretive process used by mathematics teachers in translating the intended curriculum to students. In order to achieve this objective, the study called for multiple approaches and strategies and for questions at different levels. The levels are distinct, beginning with questions asked of individual case teachers at level one, followed by questions asked of individual cases, questions asked across multiple cases, questions asked of the entire case study and ending with normative questions at level five. Yin (1984) points out that distinguishing among the levels of questions is crucial, especially when a single case is part of a multiple-case study. The different levels of questions also imply a hierarchy, meaning that a level four question could not be answered until the questions at the previous levels are answered. Furthermore, the levels imply that a cross-case question cannot be asked of an individual teacher. For example, a cross-case question may be whether there are similarities in instructional approaches across the cases. This question can only be answered when all data from the participating case teachers are available.

The current study has five different levels of questions and the answers obtained to the questions at the previous level always serve as the data base or sources of evidence for next higher level questions. The levels of the questions and the corresponding sources of evidence are shown below:

Level 1: Questions asked of individual case teachers.**Sources of evidence:**

Classroom observations involved a structured activity during which the researcher recorded observations in an operating Math 30 classroom with a focus on the physical environment, teaching strategies, and the extent to which problem solving and mathematical applications are incorporated in the lesson. The data gathered were used to answer the following questions.

1. What is the physical environment of the mathematics classrooms in terms of characteristics like size, layout, windows, blackboards, tackboards, screens, equipment, shelves, displays, furniture and mathematical resources?
2. What teaching strategies are being used as they relate to the teacher activities, student activities, use of textual resources, mathematical content, nature of the students and overall climate in the classroom?
3. To what extent is problem solving present in the classroom, based on the defined observable behaviors relative to the four problem solving indicators?

In addition, the data are used to describe the perceptions of each teacher relative to school setting, school mathematics, teaching and students.

Situational interviews occurred either during the classroom observation (when it was not disruptive to the teacher and/or student) or immediately following the class period. The questions posed arose out of a particular situation or event observed. The situational interviews differed from teacher to teacher. The data gathered were analyzed and reported for the case teachers relative to their perceptions of the school setting, school mathematics, teaching and students.

Individual teacher interviews were conducted immediately following the completion of the trigonometry unit. It was a focused interview that followed a set of questions. All teacher interviews were tape-recorded and transcribed. The purpose of this interview was to corroborate certain facts revealed by other

instruments. The interview posed the same thirteen questions to all case teachers. They are as follows:

1. What do you think of the Math 30 curriculum or more specifically the trigonometry unit in general?
2. What parts of the trigonometry unit should be omitted?
3. Should the content of the trigonometry unit be expanded?
4. How useful are the various resources (eg. texts, supplementary texts, manipulative materials, personally developed materials, etc.)?
5. What is the nature of the students and how does it affect your interpretation of the curriculum materials?
6. What teaching strategies do you use and what factors determine your teaching strategies.
7. Do you try to emphasize problem solving and mathematical applications as intended in the curriculum guide? Why? Why not?
8. What is the nature of your student evaluation (tests, quizzes, homework, worksheets, etc.)? Why do you select this particular approach?
9. What do you do by way of electives?
10. Do you review the topics taught with the students at the end of the course? What is the nature of the review?
11. What are some of the most critical factors affecting your teaching of Math 30 (trigonometry)? Why?
12. What specific concepts (skills) do you consider most important for your students to know? Why?
13. What concepts (skills) in trigonometry present students with the greatest number of problems? Why?

The responses to these questions are analyzed to determine the teachers' perceptions relative to the four curriculum commonplaces: school

setting, school mathematics, teaching and students.

Individual CPTT responses are compiled for the case teachers and analyzed with respect to their perception of school setting, school mathematics, teaching and students.

Level 2: Questions asked of the individual case.

Sources of evidence:

Class summaries of ATMT were prepared and the results are included in the case reports for each teacher. The test has three sections: (1) Mathematics in School, (2) Mathematics as a Process, and (3) Mathematics and the Student and it seeks answers to the following questions:

1. What degree of importance do students attach to mathematics in school?
2. Do students like mathematics in school?
3. What are the students' views about mathematics as a process?
4. What are the personal feelings of students about themselves and toward mathematics?

Class summaries of MDE are prepared with respect to the students' performance in trigonometry alone and for the whole course. The findings are reported in the case report for each teacher. The reason for including these data is to examine the relationship between the teacher-interpreted curriculum and the attained curriculum. The specific question that was raised in connection with these data is:

1. What is the level of achievement attained in the trigonometry unit associated with the teacher's interpretation of the intended curriculum?

Summary of case reports are prepared for each teacher drawing from the data obtained through classroom observations, situational interviews, individual teacher interviews, individual CPTT responses, ATMT and MDE. These summary reports are reported in Chapter IV and they are used to answer the

following major research questions at the case level.

- Research Question 1: What interpretation do teachers give of the intended curriculum as represented by the trigonometry unit of the Mathematics 30 curriculum? (TI questions #6, 7 & 12; CPTT survey: parts 2, 3, 4, 5, 6, & 10).
- Research Question 2: What are the classroom practices of teachers and to what extent are these practices successful? (TI questions #2, 3, 4, 6, 8, 9 & 10; CPTT survey: parts 4, 7, 9 & 10; ATMT and MDE results)
- Research Question 3: What are some of the factors which influence the teachers' interpretation of the trigonometry unit to students? (TI questions #1, 5, 6, 11, & 13; CPTT survey: parts 1, 2, 5 & 8).

Answers to these three major research questions are contained in Chapter V.

Summary of CPTT responses of survey teachers includes the responses of six Math 30 teachers from six different geographic regions across the province. The summary is contained in Chapter IV and it provides an answer to the following question.

1. What are the provincial trends with regard to the four curriculum commonplaces: school setting, school mathematics, teaching and students?

Chapter IV contains the three case study reports and the teacher sample report.

Level 3: Questions asked of the findings across multiple cases and in relation to the provincial sample.

Sources of evidence:

Individual case reports and the composite CPTT report are used as sources of evidence to answer cross-case and case-sample questions. The data compiled at the case level are subjected to further exploration and examination to identify the relationship between various factors that affect the

teachers' interpretation of curriculum materials. The fourth major research question raised is:

Research Question 4.: What is the relationship between the various factors identified in this study that affect the teachers' interpretation of curriculum materials?

The purpose of selecting the sample of six mathematics teachers from six geographic regions across the province is to situate the case teachers in the context of other practicing mathematics teachers teaching the same curriculum. The composite CPTT report of six mathematics teachers is further examined to respond to the following major question of interest:

1. How do the individual cases compare with the provincial sample of six teachers in relation to the major factors of influence on curriculum interpretation?

The answers to these questions are contained in Chapter VI.

Level 4: Questions asked of the entire Case Study.

Sources of evidence:

Multiple case report and research literature are used to answer the fifth major research question:

Research Question 5: What is the nature of teachers' functional paradigm as characterized by their interpretation of the Mathematics 30 curriculum?

The answer to the question is contained in Chapter VII.

Level 5: Normative questions about recommendations and conclusions.

Sources of evidence:

Completed case study report is used to pose two broad questions:

1. What recommendations or questions for further research on curriculum interpretation by teachers can be raised for further study?
2. What are some of the conclusions relative to the findings reached in this study?

The answers to these two questions are contained in Chapter VII.

Data Analysis

Qualitative data: These were generated from a number of sources in this study. Notes made during the classroom observations focused on the classroom setting, teaching strategies and level of problem solving activity.

The extended teacher interview with each of the three teachers was tape-recorded and a verbatim transcript was typed into a computer file. The stenographic transcripts were reviewed and found to be accurate. Speech hesitations were not identified, but all other expressions were left in the transcript. Run-on sentences were broken down to shorter sentences in order to make them easier to read.

The interview transcripts were then examined to find recurring themes. (Rist, 1982) To facilitate the process, the teacher comments were categorized using four curriculum commonplaces: (Schwab, 1971) school setting, school mathematics, teaching and students. Each of these categories was further examined to identify themes. To ensure the existence of a theme required a constant checking back into the original text and context in which it was found. The search for recurring themes was guided by key words such as: mathematics, student, teaching, diploma exams, time and problem solving.

A copy of the transcript and a summary of the main themes that emerged was sent to each teacher with the request to read it and provide feedback on the accuracy of the information and on the validity of the researcher's statements about how the teachers interpreted the curriculum materials in their classes. (Dawson, 1979) The summaries which were sent to the teachers identified

elements in the practical knowledge of the teachers which influenced the way in which they translated curriculum materials into practice in their classrooms. Each teacher responded with interest. The feedback included comments on typing errors, a few minor factual errors describing the school setting and revised passages in the transcript of what they really wanted to say to a particular interview question. The three teachers confirmed that the summary reports and statements about their interpretation of curriculum materials were valid.

Quantitative Data: The need to compute statistics based on quantitative data applied only to the administration of the CPTT survey to the provincial teacher sample and to the ATMT administration. Since the sample size ($n=6$) was small, several parts of the CPTT survey were compiled manually. For other parts of the CPTT survey and the ATMT, where the mean and standard deviation of the ratings were required, the SPSS-X program (Statistical Package for the Social Sciences) was used to do the calculations.

Statistical analyses beyond frequency counts, mean ratings, standard deviations and correlations were not seen as necessary nor sensible, given the small sample size.

The results from the MDE were statistically analyzed by Alberta Education and received in useable form.

Analytic Strategies

The nature of the five broad research questions and the lack of statistical strategies for the small data points in this study made it necessary to use other analytic techniques to analyze the data. Yin (1984) points out that analyzing case study evidence is especially difficult because the strategies and techniques have not been well defined in the past. He suggests two general strategies, relying on theoretical propositions and developing a case description, which underlie specific analytic procedures.

The current study followed theoretical propositions as the general strategy, because the general objectives and design of this case study were based on such propositions, which in turn reflected the major research questions. The propositions and questions also shaped the data collection plan and the development of the research instruments. Within this general strategy, pattern-matching was the most prevalent mode of analysis used in this study. Observed patterns were compared with predicted or theoretical ones to determine if there was a match. If the match was a "good" match or alternative patterns of predicted values were not found, strong inferences were made. If the match was confirmed over the three cases, literal replication of the single case was achieved, which made the cross-case results even stronger. In the absence of statistical techniques, the researcher had to use some interpretative discretion in deciding a pattern had been matched or mismatched.

Two "lesser" modes of analysis were also used in connection with the current study. These are analysis of embedded units of analysis and repeated observations. The embedded units in this study were the ATMT and MDE results. The analysis of embedded units was done first, which reflected the propositions for these units and not those for the larger case. The results were interpreted at the single-case level and served as factors in a pattern-matching analysis at the single-case level. The patterns for each single case were ultimately compared across cases, following the replication mode for multiple cases. Conclusions drawn for the multiple cases became the conclusions for the overall study.

Repeated observations were used in this study in two ways: (1) repeated "sites" (eg. 3 single cases) and (2) multiple instruments to gather data about the same component under study. Both ways assisted in corroborating the findings and strengthening external validity.

F. Criteria for Interpreting the Findings

There is to date no precise way of setting the criteria for interpreting the findings in relation to a predicted or theoretical pattern. (Yin, 1984) Various analytical methods, such as categorizing information; tabulating the frequency of different events; and calculating mean ratings and variances, assisted in determining an observed and predicted pattern matched or if two potential propositions were rival propositions. This kind of interpretation is highly insensitive to very subtle patterns and as a result the researcher did not postulate these subtle patterns, but rather focused on patterns that were sufficiently contrasting or led to gross matches to draw a conclusion.

The identification of specific criteria for interpreting the findings in this study was done during the data analysis steps because the researcher needed to "play" with the data in order to understand the findings and attach meaning to them.

Criteria for Judging the Quality of this Research Design

The research literature identifies four quality tests for case studies (Yin, 1984) and they were discussed in Chapter II. In order to achieve each of these quality tests for the current study, a number of tactics were employed throughout the study.

Construct validity is the first quality test and it was achieved by defining the areas or specific changes that were to be studied and by demonstrating that the measures of these two areas or changes were indeed those that had been selected. (Yin, 1984) In the context of the current study, it was the interpretive process of teachers in translating curriculum materials to students that was the area of study. The various instruments and strategies that were employed in the current study, were designed to ensure that each one of them was indeed measuring an aspect of the interpretive process and that in total they would produce a measure of the interpretive process used by

teachers. To ensure that this multiple case study had construct validity, three tactics were incorporated. **First**, the use of multiple sources of evidence was used with the aim of developing convergent lines of inquiry. This included teacher observations, interviews, surveys, and tests - all of which provided multiple measures of the same phenomenon. **Second**, a chain of evidence was established which allows the reader to follow the derivation of any evidence from the initial research questions to the ultimate case study conclusions. While important in increasing construct validity, the maintenance of this chain of evidence also increased the reliability of the information in this case study. The **third** tactic used in this study was a review of the draft case study report by key informants. The case study teachers in this study were the key informants. This tactic was seen as a way of corroborating the essential facts and evidence presented (Schatzman & Strauss, 1973) and thus increasing the construct validity.

Internal validity, as a second quality test of this study, is of some concern in answering research question 3. The answer to this question involved showing a causal relationship between an event and some proposition. Two approaches were used to deal with causal relationships in this study. One, descriptive data were quantified and used in a causal sense to explain a relationship. The other, is pattern-matching which involved a comparison between an observed pattern and a predicted pattern. Both analytic strategies helped strengthening the internal validity of this study. Two other modes of analysis were used that strengthened the internal validity. These included analysis of embedded units and repeated observation across the cases and multiple instruments within the cases.

The third quality test that was applied to this study is **external validity**. This test required establishing the domain to which the study's findings could be generalized. Unlike survey research where generalization from correctly selected samples to a larger universe occurs, case studies are limited to generalizing findings to "theory". Therefore, the findings from one case in the current study are not generalizable to the next case, the provincial sample or

the mathematics teacher population. This is because case studies rely on analytical generalization. The findings of the individual case studies within the current study were generalized to some broader theory which was tested through replication of the findings in the other two case studies. The current study, therefore, included the use of replication logic in a multiple case design to increase external validity.

The fourth quality test that was applied to the current study is **reliability**. The goal of reliability is to minimize errors and biases in a study. Two case study tactics were identified (Schatzman & Strauss, 1973; Yin, 1984) to achieve this goal. The use of a case study protocol was prepared that contained the instruments, questions, procedures and general rules which would enable someone to repeat the study with the same results. The second tactic used was the development of a case study data base, which is also connected to the notion of maintaining a chain of evidence. The general approach to increasing reliability of the current study was to make as many steps as possible as operational as possible.

IV. THE CASES AND THEIR CONTEXT

This chapter contains three major sections. In the first section the main elements of the Alberta Mathematics Curriculum are presented, critically analyzed and related to the current study. This is followed by the case study reports which incorporate the findings from the classroom observations, situational interviews, teacher interviews, individual CPTT responses, the Attitude Toward Mathematics Test (ATMT) and the results from the Mathematics Diploma Exam (MDE). The third section contains a summary of the six sample teachers who were selected from six regions across the province and who responded to the CPTT.

A. Alberta Mathematics Curriculum

Since the Math 30 program, or more specifically the trigonometry unit, is the focus of the current study, a brief outline of the program, a critical analysis of the program of studies and the relationship of the intended curriculum to the teacher interpreted curriculum are presented here.

In matters of curriculum the Minister of Education, through the School Act (1980), Section 11, is granted the authority and responsibility to prescribe programs, courses of study and instructional materials for grades 1 to 12 in the schools of Alberta. A number of service documents, containing relevant topics that range from goals to specific content for each course, are made available to those involved. Specific reference to these documents will be limited to the Junior-Senior High School Handbook (1985-86), the Program of Studies for Senior High Schools (1978) and the Mathematics 10-20-30 Curriculum Guide (1983). The Junior-Senior High School Handbook as well as the Program of Studies for Senior High Schools begin with a statement of the goals of basic education for Alberta and they are divided into goals of schooling and goals of education. The goals of schooling refer specifically to the responsibilities to be assumed by the schools, while the goals of education are viewed as a shared

responsibility between the school and the community. Further information contained in the handbook deals with types of high schools, course numbering system, time-credit relationship (ie. 25 hours equals one credit), special circumstances and an actual list of all high school courses.

In addition to the goals of basic education in Alberta, the Program of Studies for Senior High Schools 1978 (Alberta Education, 1978) lists the objectives for each senior high school course and the authorized texts.

The curriculum guide for Math10-20-30 is a more comprehensive document, containing the program rationale, program structure, program emphasis, program planning, the elective component, suggested time allocations, suggested elective outlines, learning resources, goals of the senior high school mathematics program and the objectives for each mathematics program. As well, the guide offers some advice on how to teach problem solving and deal with the academically talented.

The remainder of this discussion will deal primarily with Math 30 as presented in the curriculum guide. Of the three streams of mathematics programs offered at the high school level in Alberta, the Math10-20-30 program is clearly intended for academic students seeking admission to university or other post-secondary institutions.

As alluded to earlier, a new Math 30 program was introduced in September 1983 and along with it a mandatory provincially administered diploma examination that students are required to write at the completion of this course.

An examination of the Math 30 curriculum guide reveals that some significant changes have occurred. The changes are not just related to content but also related to the perspective of a number of curriculum components.

The program rationale (p. 3), which provides the philosophical framework, contains six major statements:

1. Mathematics is seen as an important component of public education and essential in order to function effectively in today's society.
2. Mathematics as a formal study of mathematical concepts and relationships must be in balance with applied use of mathematics in everyday situations.
3. Mathematics as a part of the basic education of students at all levels must ensure continuity and yet be responsive to emerging influences that are characteristic of our changing times.
4. Mathematics programs should have some flexibility in the selection of topics to meet the needs of the students.
5. Mathematics programs should integrate the use of calculators and computers in the instructional process.
6. Mathematics programs should emphasize problem solving, integration of microelectronic technology and applications.

These statements have been echoed by a number of other writers (Robitaille & Dirks, 1982; Usiskin, 1980; NCTM: An Agenda for Action, 1980; NCTM, PRISM, 1981; NCTM, 1989) and appear to be the current thinking among mathematics educators.

Given this kind of rationale as the philosophical basis for Math 30, what kind of goals does it generate? The Math 30 curriculum guide expresses the goals in relation to three main expectations and needs: those of the individual, those of the discipline of mathematics and those of society at large. (p.18)

With regard to the students, the curriculum guide seems to view Math 30 students as academically capable, not in need of any remediation. This observation is supported by the statement in the curriculum guide on "Provision for the Academically Talented". (p.11) It suggests that "...topics within the elective component of the course should be utilized to challenge the academically talented students." (p.11) It goes on to further suggest that "...teachers may also extend core topics and concepts to a higher level of complexity to meet the needs of the stronger academic student." (p.11)

The fact that there are three different streams of mathematics at the high school level suggests that Math 30, as the academically most demanding, is not intended to appeal to all grade 12 students. Its general nature is an emphasis on "theoretical development of mathematics concepts and relationships through deductive reasoning." (p.5)

The goal statements for mathematics seem to imply a broad range of expectations of teachers. At one extreme of the continuum the goals are typically utilitarian. An example of this would be:

To develop [in students] the ability to utilize mathematical concepts, skills and processes. (p.18 bracketed comment added by this writer)

The other extreme of the continuum deals with goals that are typically cultural and historical in nature. The following are excerpts that illustrate this point:

To develop an appreciation [in students] of the contribution of mathematics to the progress of civilization.

To develop a mathematical competence in students in order to function as citizens in today's society.

To develop an appreciation [in students] of the importance and relevance of mathematics as part of the cultural heritage that assist people to utilize relationships that influence their environment.

To develop an appreciation [in students] of the role of mathematics in man's total environment. (p.18, bracketed comments added by this writer)

Begle (1979) suggests that the strong influence by a large number of interest groups is responsible for such a wide range of goals. Since unanimous agreement on the choice of goals is often impossible or not even expected, inclusion of all suggested goals is the result. It would appear that under those circumstances teachers would find it very difficult to link the goals with the actual objectives.

Examining the Math 30 curriculum in some detail reveals this linking

problem at once. The objectives are entirely content related and their logical or pedagogical connection with the goals is not completely clear. In some instances the Math 30 objectives parallel the goal statements, but are stated more precisely and specifically. For instance, it could be argued that the objective:

Derive and apply the Pythagorean relations:

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta \quad (\text{p. 70})$$

is closely linked to the goals:

To develop the ability to utilize mathematical concepts, skills and processes, and/or

To develop the power of logical analysis and inquiry, and/or

To develop an ability to communicate mathematical ideas clearly and correctly to others. (p.18)

However, it is not clear how any of the content objectives ensure achievement of the goal dealing with the development of positive student attitudes towards mathematics. Also, it is difficult to determine whether any of the objectives lead to an appreciation of the role of mathematics in man's total environment. The fault may not lie so much with the goals than with the manner in which students are taught or evaluated. The provincially-administered mathematics diploma examination is exclusively content and skill oriented with no provision for measuring or assessing the students' attitude towards mathematics or the students' appreciation of the contribution of mathematics to the progress of civilization, to name two. It is not even clear that the students' understanding of the common unifying structure in mathematics is being assessed. The implication here is that, in actual practice, teachers are not required to deal with those goals that are not evaluated. The end result is that teachers, as Wilson (1971) puts it, operate their mathematics class just to cover the material in the textbook or they think of the goals of instruction only in terms of content. (p.650)

The curriculum guide makes several suggestions as to how teachers should organize and present learning experiences for students. Teachers are to emphasize problem solving, integration of microelectronic technology, applications and the teaching of statistics using the experimental approach.

A disparaging account of traditional mathematics is given in the curriculum guide, saying that:

Typically, the problems presented to students in the past required nothing more than a repetition of operations involving a series of computations and, in many cases, number transfer in stated formulas. The solution process was largely mechanical in nature once the steps needed to arrive at an answer were known (or stated) and subsequently practiced by the student. (p.6)

However, the curriculum guide is silent on the question of how teachers should proceed with the acquisition of these new skills, unless it is assumed that teachers possess these skills already but simply do not use them.

The curriculum guide lists two prescribed references: Math 1s/6 and Foundations of Mathematics For Tomorrow: Senior. In addition, Holt Math 6 is available as a recommended reference. While case teachers feel that the two prescribed references provide reasonable coverage of the concepts and skills, they do not think that they deal sufficiently with problem solving and mathematical applications.

Earlier on it is stated that the curriculum guide views Math 30 students as academically capable of handling this course. The curriculum guide does recognize, however, that a range of abilities may exist in any given Math 30 class. The elective component of the program is intended to deal with these differences. It allows for variety and flexibility in the choice of topics covered within the stated guidelines.

The elective component primarily offers an opportunity for students to spend time on interesting and useful areas of mathematics not necessarily contained in the core and independent core components. (p.9)

The elective component of the program may be:

1. a content area not prescribed as a core or independent core topic.
or
2. a locally developed unit as determined by the teacher or school system.
or
3. an extension of the subject matter in any of the core or independent core topics to provide students with enrichment.

It is interesting to note that while the elective component is mandatory, the curriculum guide merely suggests that it be included in the evaluation of students at the school level only. The diploma examination does not deal with content from any of the elective topics. This being the case, and recognizing that teachers have needed the total time available just to cover the core topics, suggests the possibility that not all classes study elective topics. The only way to correct this situation is by addressing, what Hall and Loucks (1982) call, "...the fundamental gap between policy initiatives and the realities of life in schools." (p.134)

The review and critical analysis of the mathematics program suggest that there is a linking problem between the rationale, goals and specific learning objectives. The current study is examining the link between the intended curriculum and the teacher- interpreted curriculum. Given that there is a linking problem within the intended curriculum, what interpretation do teachers give of the intended curriculum? What are the classroom practices of teachers and to what extent are these practices successful? What elements of the intended curriculum are being implemented by teachers? On the one hand, teachers must prepare students for the MDE which is skill oriented and, on the other hand, the goals demand that teachers develop an appreciation in students of the contributions of mathematics to the progress of civilization, which is not reflected on the diploma examination. Are teachers focusing on skill oriented objectives instead of devoting time to cultural or historical objectives like dealing with the history of trigonometry? The current study is examining these issues.

Given that there is a new curriculum with rationale, goals and specific learning objectives, what are the characteristics of the teachers' functional paradigm within the context of this new curriculum? The current study will lead to an identification of the characteristics of the teachers' functional paradigm and focus on explanatory principles that address the question why teachers function in particular ways.

B. Three Mathematics Teachers

In this section the three case studies are reported with a focus on the first three research questions and the four curriculum commonplaces: school setting, school mathematics, teaching and students. (Schwab, 1971) Multiple measures of the same phenomenon provided multiple sources of evidence, enabling the researcher to develop convergent lines of inquiry. Furthermore, this allowed confirmation of the findings through triangulation.

The names of the teachers; Alice, Bill and Carl; are fictitious, as are the names of the schools; School A, B and C. However, the people, places and situations are real, and care was taken to describe them as they are.

Alice - the curriculum deliverer

Alice is a mathematics teacher with almost thirteen years of teaching experience, two of which she taught high school mathematics. She is currently teaching three mathematics courses at the grade 10, 11 and 12 level. Her background includes seven mathematics courses and two methods courses in mathematics. Alice describes her background for teaching mathematics problem-solving and applications as "satisfactory". However, she deems her background for implementing the overall philosophy, goals and objectives of the course to be "good".

Alice's School Setting

Alice teaches in School A which has a total enrollment of approximately 450 students ranging from grade ten to twelve. School A also offers home economics and industrial arts to students from a neighboring junior high school. The school is seventy percent utilized with sixty percent of the students residing within the community and forty percent being bused from the surrounding areas. There is a total staff of twenty six teachers, four being involved in teaching mathematics on a full-time basis. This school offers all three streams of mathematics plus Math 31. Nearly all programs are semestered. School A offers a broad program which includes: academic, business education, fine arts (band, drama & art), home economics and industrial arts and work experience.

School A is a single storey masonry structure which was build in 1968. It is spacious, has wide hallways and is very well maintained. Many of the hallways are decorated with murals painted by students. The community in which this school is located has approximately 7500 residents and almost again as many residing in the surrounding area. Major employers in the area include provincial and federal agencies, the oil and gas industry, forestry, mining, service industry, railway and the school system. There are also a significant number of self-employed professional people residing in the community. The residents outside the community are either self-employed farmers or they are employed in the areas identified above.

The community has extensive recreational facilities, a public library, hospital (active treatment and nursing home), senior citizen's lodge and a post-secondary education facility.

The school setting and all its related factors emerged as a major influence on Alice's interpretation of curriculum materials for use in her classroom. While the influence of individual factors differs, collectively they have a powerful impact on her interpretation.

In general, Alice is satisfied with the classroom in which she teaches mathematics, but she is not so satisfied with the limited availability of computers.

Currently, computers are only available for the lesson on graphing and Alice would prefer having computers on a permanent basis in her classroom. She is of the opinion that computers not only increase her efficiency, but they also lead to a better understanding of the mathematical content and to a greater enjoyment by students. Alice feels that computers are the way of the future and that students must be exposed to this technology. In addition, she wants to use computers to tackle interesting problems that she otherwise would not consider doing. Calculators, which are used by her students at all times, are of considerable value in her opinion, but they do not allow her the extensive use that a computer affords. For one thing, students have a variety of different calculators not all having the same capabilities and furthermore, none of the calculators has graphing capabilities. Alice feels that calculators and computers are essential in the classroom, particularly as they relate to problem solving. Without computers, she feels, she is not able to deal with a number of good application problems.

The central importance of problem solving in the curriculum worries Alice somewhat for several reasons. Although she agrees with its importance, she is not quite sure if she really understands what "authentic problem solving" is. As a result she selects all of her application problems from her own collection of problems which she built up over the years and some from the text. However, the text, Math Is/6, is a constant source of frustration to her. She points out that aside from the inadequate coverage of many topics and frequent mistakes, the text is seen by Alice as being woefully inadequate as a problem-solving resource. Given the importance of problem solving in the curriculum, she feels that the curriculum should be matched with appropriate and adequate textual resources. She points out that if it was not for her good background in mathematics and her own collection of problems, she would have serious difficulties in addressing the problem-solving objective of the curriculum. In connection with problem solving, Alice makes it quite clear that unless appropriate resources are readily available to the teacher, there will not be the variety or extent of problem solving in mathematics classes. It is obvious from

all the data that Alice values a good text because she relies on it extensively.

Another school setting factor with considerable influence on Alice's interpretation of curriculum materials is class size. She says: "...if you are working with the average run of the mill sort of Mathematics 30 class where you have a fairly large number of people who have to drill on skills, then that's where a lot of the time and energy goes and they seem to be the people who get the attention; I suppose justifiably so. You tend to teach to the middle of the group and as long as the groups are full of different ability levels, I find it hard to come up with a problem situation and to propose it and to have any success in people solving it because the majority of the people are still working back at level one where they are learning how to apply the very first basic skill." Alice clearly holds the class size responsible for her "middle of the road" approach and her emphasis on drill and practice.

Alice is quite aware of the expectations that the parents or the community have of her. In fact, these community expectations and pressures from parents that students do well in mathematics generally and on the MDE specifically have led to the adoption of different practices in her classroom. There is an increasing emphasis on preparing students for the MDE which includes reviewing and writing previously written MDE and confining the content to the core program as much as possible. The pressure to get students to be successful on the MDE, in the school-based exams and in the course is very strong, because it determines for most of them whether they can continue with their post-secondary training.

In connection with these pressures and challenges, Alice identifies a strong mathematics background and teaching experience in mathematics as being helpful. Of course; Alice has all of that and she feels well equipped to implement the overall philosophy, goals and objectives of the course. While her professional background and experience have a strong impact on her interpretation of curriculum materials, the number of mathematics teachers, and related to that the level and quality of interaction among these teachers, is having a strong influence as well. Alice indicates that, "...I couldn't have

survived without that kind of discussion, because at least I felt there were two of us who shared an opinion". The interaction with other teachers, in Alice's case, is very positive, providing her with opportunities to check perceptions, clarify interpretations of curriculum objectives and ask for assistance.

Alice's View of School Mathematics

Alice's classroom practice reflects a view of school mathematics as a well established body of knowledge, concepts, skills and procedures. She sees the content as sequentially organized and the various parts of the mathematical structure in a logical relationship to each other. Although she is constantly building mathematical knowledge on the students' existing knowledge base, she believes strongly in mastering and maintaining basic skills. She accomplishes this through almost daily reviews of previously learned material. She points out that these reviews are done so that:

"... students see the connection between previously taught materials and new materials, so mathematics doesn't look like a bunch of individual and separate lessons to the students. It becomes a more unified whole. I want students to see each lesson as a building block of the bigger structure and that the blocks fit nicely together."

The content of a particular topic is determined by its relevancy to the curriculum and consequently Alice's program consists of the core curriculum only. Even in relation to the core program, relevance of content is defined as being part of the curriculum objectives for the core program. All sources of evidence suggest that the core program is of primary concern to Alice, yet, she chooses to extend certain topics from time to time to provide for enrichment. These extensions include such activities as proofs for law of sine and cosine, proofs of some trigonometric identities and graphing with computers. Alice is in constant search for meaningful extensions of core topics as is evident from her comment:

"I find it very difficult to do a lot (of extensions). I try to choose topics that coordinate with things I'm doing already so that we have an opportunity to develop what is already there and I find it hard to fit that in." (bracketed comment added by writer)

Her view is that whatever is covered within the elective time or the extensions should fit the core topic and lead to a better understanding of the core content. Alice judges extensions in terms of their usefulness for further study in mathematics, and she favors them if they unify basic ideas or if they are interesting examples of mathematical reasoning. She made it quite clear that she is opposed to the inclusion of unrelated topics, like the history of mathematics, because "...how do you justify this to the students who still have difficulties with the core content." Alice also cites the time factor as relevant in the selection of curriculum materials and topics, suggesting that there is barely sufficient time to do the core-related activities properly. She indicated that mathematics teachers tend to compartmentalize their teaching, not necessarily relating concepts with each other nor across the subjects. She suggests that this view is prevalent, because students tend to be "concerned only in what they have to do and learning what they have to learn". Consequently her emphasis is on learning and understanding basic skills that have relevance to the core curriculum. Core content and relevancy are synonymous for Alice. She believes that mathematics plays an important role in everyday life and that those who master the basic skills are more able to solve "real life" applications. Trigonometry is viewed by Alice as basic skill work that needs to be mastered before students are in a position to solve problems.

The most frequently cited reasons by Alice for including topics in her core program are: they are listed in the Mathematics 10-20-30 Curriculum Guide (1983) and they are useful for further study in mathematics. The usefulness of topics for further study seems to be closely related to Alice's personal list of topics that are outside the curriculum. According to the CPTT, her personal list includes formal proofs of quotient relations, reciprocal relations, Pythagorean relations, complementary arc formulas, sum formula for tangent, law of sine and

cosine and the double angle formulas for the sine and cosine function. Coverage of these personal topics is not expected by the curriculum guide. Although the observations and Alice's own comments indicate that she closely adheres to the core curriculum, there is evidence that she consciously deviates from it as well. Her personal list is included because she feels these topics "round out" the students' view of mathematics and therefore they should be included. She also mentions that they were once part of the trigonometry unit and the new curriculum no longer includes these. This phenomenon is quite common in the other cases as well where topics from previous courses are incorporated in the new curriculum. However, her deviation from the core curriculum occurs also in other areas. For example, the negative arc formulas are simply stated with informal justification, the sum and difference formulas for the sine and cosine functions are simply stated without justification and proofs are provided for the sum formula for the tangent function and the double angle formulas for sine and cosine. While in some instances Alice falls short of the curriculum expectations, in others she exceeds them. Whenever Alice exceeds the expectations, she justifies her decision by saying that this activity unifies basic ideas or that it is an interesting example of mathematical reasoning.

Alice views mathematics as a powerful problem-solving tool and she does emphasize problem-solving within the context of real life applications. However, this occurs as a separate activity which employs the skills acquired and it is confined to the text.

Alice can be characterized as a competent and confident teacher at the level of school mathematics, but the mathematical content of her course is very much influenced by her own comfort level, the MDE, by what students will need to know to succeed in their post-secondary education, by the Math 30 Curriculum Guide, the student text and whether the material builds on or extends concepts covered in previous grades. The selection of curriculum materials seems to receive little or no influence from such factors as: what students will need to know to understand the importance of mathematics in their daily life, what society needs by way of informed citizens, what students require

to understand the structure of mathematics and what students require to understand the historic development of mathematics.

Alice's Teaching of Mathematics

The classroom observations indicate that Alice manages the classroom routines, such as taking attendance, collecting and returning assignments, making relevant class announcements and reading school announcements, with considerable efficiency.

A typical mathematics lesson begins by asking students if they had any questions about or problems with homework or the class assignment. Students always raise questions and Alice demonstrates and explains the problem on the blackboard. Sometimes Alice draws the students' attention to a particular problem and discusses it in some detail to ensure understanding by all students. Nearly all students complete their homework every day and most are successful. Alice frequently approaches individual students to see if they had difficulty with homework. She is quite prepared to offer her assistance to students outside the regular classroom lesson.

During her demonstrations of homework problems there is much interaction between Alice and the students. A good variety of questions are being raised and different solution methods are being considered. Alice consciously incorporates the four problem-solving steps when illustrating examples. She seems to place particular emphasis on the "Looking Back" step, questioning the reasonableness of the answer and exploring alternative methods. Problem solving is not something that comes easy to Alice and she says: "I find it very difficult to do problem-solving the authentic way. I mean, what is a real and what is an artificial problem? I think the exercises in the text are pretty good. And, besides there isn't a lot of time to go outside and do indirect measurement or go on a field trip in search for real-life problems or applications. I am not sure that I know enough about these real-life situations to make it meaningful for my students. I like problem solving and I think it is

important but the problems must be readily available for use."

Once the homework has been dealt with, Alice proceeds to the next topic. Following some brief introductory comments, Alice normally illustrates an example which involves the students at every step of the process. During this period, Alice develops a variety of blackboard notes which are copied by the students. This part of the lesson frequently includes a review of previously taught materials. Alice also uses this opportunity to connect newly acquired knowledge, skills and concepts with those taught in previous lessons or grades so that students gain a better understanding of the "bigger picture". In most cases, Alice demonstrates several examples before students begin to work on assignments. Most assignments are selected from the Math Is/6 text and on some occasions they are supplemented with worksheets from different resources.

The lesson development phase is normally followed by student practice. Students are given an assignment from the text, a worksheet or both and they have the remainder of the class period to complete it. Students are expected to complete any unfinished work at home for the next day. Alice provides much individual assistance during the student practice time. She circulates among the students observing, asking questions and assisting those who have questions. Not only does Alice make additional time available for students with difficulties, but she also fosters a climate in her classroom that is conducive to effective learning. She prepares well thought-out lesson plans, presents lessons that are clear and systematic to students, has clear and high expectations of students, fosters group work and co-operation, asks good questions to make students think, makes the lessons interesting and above all, she has a good sense of humor.

The classroom observations as well as the other data gathering devices suggest that Alice uses a very systematic approach in her teaching. She is very task oriented and values flawless teaching performance. During one lesson she demonstrated the proof for the law of cosine and prefaced this demonstration with the comment: "You must not get married to the same set of

letters. The proof is independent of the letters chosen." As she proceeded with the proof, she ran into difficulties and became confused, forcing her to refer to the book and start all over again; this time using the same letters as shown in the book. This incident was quite embarrassing and unacceptable to Alice and it reveals just how strongly she values professional competence.

Alice's classroom practices and interpretive process are characterized by a number of values. Pedagogical efficiency is the most prominent value and it is closely linked to time. It appears to permeate every aspect of Alice's teaching and her comments refer repeatedly to the time factor. The time factor is considered during the planning stages of her lessons; in selecting a teaching strategy, like using computers for graphing; in the selection of problem-solving activities; in administering tests and quizzes; in selecting curriculum materials and in defining the boundaries of her core program. These time concerns are illustrated in a number of statements made by Alice.

In relation to teaching strategies, specifically her use of computer she says:

"... They also save us a lot of time in teaching and so we can do better things with that time." or

"In the past it took me an 80-minute period to graph the sine function on the blackboard and now I can explore all six trig functions plus all changes in the parameters of these functions in an 80-minute period."

Her teaching strategies in general reflect her concern for time. The use of lecture-response, demonstrations, class discussions, drill and practice and checking homework are prevalent in her class. Teaching strategies that are viewed as being inefficient in terms of time are never or rarely used.

In relation to administering tests and quizzes she says:

"... You can't administer too many quizzes or tests because of time. I need to stick closely to the objectives." or

"A quiz is an effective and efficient way to zero in on a specific set of skills or a concept and get immediate feedback. I can normally mark them the same day or the latest by the next day and return them."

In relation to selecting curriculum materials she says:

"... I have to add time as an important factor, too. You know time is very critical."

In relation to defining the boundaries of the core program she says:

"...If you want to do a good job of the core program, that takes nearly all the time you've got. Then you want to review a bit prior to the finals and that is it. As a result, anything that is not in the curriculum will not be included in my lesson."

While Alice claims that anything that is not in the curriculum is not included in her lesson, she is clearly not implying that all things contained in the curriculum are included in her lesson. This is particularly the case with the mandatory elective component which is not emphasized by Alice at all. When asked why the elective component is not emphasized, she responded that the material is not on the MDE and therefore the time is spent in areas where it counts.

The influence of the MDE on Alice's interpretive process is evident from all data gathering devices. As a result of this influence, Alice places considerable value on student understanding and success on the MDE. This is quite consistent with the observation of MacDonald and Leithwood (1982), that a major factor in curriculum decision making is the teachers' overriding need for "achievement", defined in terms of assisting the students to become more cognitively competent. Student understanding is a central theme for Alice and the main purpose of her teaching. Closely connected with her desire to see students cognitively competent is her preoccupation with the students' success on the MDE which is seen to some extent as a measure of the students' cognitive competence. The strong impact of the MDE on Alice's planning, curriculum material selection and general classroom practice is evident by several of her statements and actions. Alice constantly reviews, connecting current knowledge with previously taught materials and using previously written MDE for practice.

Alice's teaching also reflects the value of academic rigor in that any deficiencies in the textual resources is compensated for through supplementary resources, either from other texts or prepared by herself. Academic rigor is also evident along the mathematical content dimension. Her high expectations with regard to proofs for the law of sine and cosine and some trigonometric identities is evidence of that. Alice is of the opinion that these proofs are important and that it is not enough that students just recall identities and then apply them appropriately. When it was pointed out that the curriculum expects students to merely apply them, Alice responded by saying:

"... Students need some exposure to the nature of proof. This will give them some appreciation of the type of reasoning required in conducting mathematical proof. This is a good example of what I mean by extending the core program. I use my elective time for this."

Despite Alice's high expectations of students, her values of pedagogical efficiency and academic rigor and preoccupation with the MDE, motivating students is of considerable importance to her. She is very conscious in selecting activities that are interesting to and enjoyed by students. The use of computers for graphing, the extensions of core topics and the selection of particular teaching approaches are all chosen to hold the interest, make it enjoyable and consequently motivate students. The availability of computers on a limited basis is frustrating to Alice. She is hopeful that eventually she will have computers in her classroom on a permanent basis for the purpose of enriching the students' program and solving interesting problems.

Alice is in many respects a "traditional" teacher. She teaches primarily in whole-class mode, but is also sensitive to the needs of individuals by setting the students to work in small groups when appropriate. This depends very much on the specific lesson. Her personal style of interacting with the students indicates that she values their perspective and their participation in the class activities. Despite her easy-going style, she has complete control of the class and she is perfectly capable and prepared to admonish if necessary. Her classroom is

managed in a business-like fashion with nearly all students on task at all times. There is clearly structure in Alice's classroom which is a dominant feature of her pedagogical framework. She feels that this structure is externally imposed by the existence of a clearly defined core curriculum and the requirement to write the MDE. Within this structure, Alice tends to return to approaches which have worked for her well in the past and they have a strong influence on her selection of curriculum materials for use in her classroom. In addition, she identifies her own interest, background and preference as a strong influence in making curriculum decisions. Alice experiences some tension with respect to her pedagogy: on the one hand she wants students to understand the mathematical content and succeed on the MDE, and on the other hand she would like to engage students in exploratory activities with due regard for their individual needs. The first dimension of her pedagogy implies structure with all students achieving the same objectives and mastering the same content, while the second dimension implies an openness and a recognition of individual differences. Alice has somewhat reconciled this tension by choosing a "middle of the road" approach, offering the same content to all students and providing extra help to students who experience difficulties.

Alice's Students

Alice has good rapport with her students and she enjoys working with them. Although the students in this course are streamed based on their mathematical background, Alice finds that there is still a wide range of mathematical abilities and interests. As a result, much of her time is spent on building relationships with her students that have a focus on the students' abilities and interests. Despite the wide-ranging abilities among her students, she believes that all of her students are capable of handling the demanding academic work. She gears all of her energy and effort toward ensuring that students succeed in her course. This is best revealed by the extent to which Alice reflects on her pedagogy during the planning stage, which results in the

selection of particular teaching strategies, content and student activities. Alice seems quite confident that all of her students have a good level of understanding of the trigonometry unit, indicating that even the weaker students should be able to pass this unit. This is an indication not only of her confidence in students, but also of her own effectiveness as a teacher. The difficulties experienced by some students are, in her opinion, not so much a question of ability but rather a question attitude. Whatever the difficulties experienced by students, Alice is quite prepared to offer assistance either during classtime or outside class. She sees program modification for students with difficulties not as a viable alternative, because all students are required to write the MDE at the completion of the course. Even the weaker students must master the skills if they wish to be successful on this exam.

Among the different approaches used by Alice to address the varying abilities and interests of her students, additional practice and explanations of the same material is the most frequently used approach. In terms of addressing students' interests, these are most often accommodated through extensions of core topics, inclusion of challenging problems and the use of computers.

Alice's focus on students' understanding, ability and interest characterizes her relationship with her students. The inclusion of topics and her treatment of these in the classroom are strongly influenced by these student factors. This is repeatedly confirmed during the classroom observations and the CPTT.

In response to the question, of what other student factors affect your teaching of trigonometry, Alice responded: "...it depends on the groups that I have or my perception of them and the kind of background that they have". Inherent in this response is a statement about the students' attitude and academic ability. Although Alice describes the attitude of her students toward learning mathematics as "very good", the student responses on the ATMT do not confirm her perception. The responses suggest that her students attach the greatest importance to memorizing trigonometric formulas and theorems, checking answers to a problem by going over it and solving trigonometric application problems. The importance attached to these activities by students is

consistent with Alice's classroom practice. She emphasizes these activities because they enhance the students' skills in writing the MDE and consequently students attach a high degree of importance to them. However, the degree to which students like mathematics in school is only minimally related to the degree of importance that they attach to it. For example, memorizing trigonometric formulas and theorems is very important to students, but they do not like this activity. The student responses suggest that the opposite is true as well. For instance, students attach a low importance to using electronic devices for solving problems in trigonometry, but they certainly like it. In general, Alice's students rate the importance of mathematics in school higher than the degree to which they like it. The students' views related to Mathematics as a Process suggest that they see trigonometry as a set of rules that can be followed when solving problems. Students feel that trigonometry helps them to think logically and in accordance with strict rules and there always seems to be different ways to solve most trigonometric problems. The students' personal feelings toward mathematics suggest that they feel good about solving mathematics problems by themselves, that they really want to do well in trigonometry and that they usually understand what is being talked about in trigonometry. These attitudinal findings are consistent with the classroom observations and they are a reflection of Alice's overall approach to teaching mathematics.

The students in Alice's class are generally well-behaved and industrious. She is very conscious in maintaining a productive environment which she achieves through her classroom structure, the nature of activities and her expectations. Alice's overriding concern is that students understand mathematics and succeed on the MDE. However, the students' future careers and their post-secondary education choices have a strong influence on her curriculum material selection as well.

Alice describes the level of understanding of her students in trigonometry as "good". However, this is not borne out from the results on the MDE. With respect to the students' performance in trigonometry alone, they are well below (59%) the provincial average (64%), however the overall achievement for

Mathematics 30 for her class (64%) exceeds the provincial average (63%) slightly. Alice points out that her students are very exam-oriented, showing only interest in content and skills that are anticipated to be on the MDE. She expresses some frustration over this and wonders how she can deal with this. After some reflection she concludes that, for a start, she needs to place less emphasis on the MDE herself. She concludes that her continual reference to the exam and her use of previously written MDE for practice purposes need to be examined.

Summary Statement of Alice's Case

Alice is committed to teaching mathematics with an emphasis on student understanding. Her view of mathematics is closely connected to her manner of teaching it. Alice builds the students' mathematical knowledge on their existing knowledge and through a strong emphasis on mastering basic skills. Her overriding goal, however, is that students see the connection between the concepts and skills and that they ultimately come to view mathematics as a unified whole.

Alice's teaching is somewhat caught in a tension, in that she genuinely wants to engage in exploratory problem-solving activities with computers on the one hand and on the other hand she consciously limits the content to the core curriculum with occasional excursions into her personal list of topics. This tension is clearly the result of students having to write the MDE. She seems to manage this tension quite well by integrating both the structured sequential approach and the problem-solving approach into the operation of her classroom. Alice holds a number of strong beliefs and values with respect to school setting, school mathematics, teaching and students and for the most part she is quite aware of their influence on her interpretation of curriculum materials for use in her classroom.

Bill - the process builder

Bill is the assistant principal of School B and one of the mathematics teachers who normally teaches Math 30. He has taught mathematics for all of his eleven years of teaching experience. His current assignment includes grade 8 mathematics and Math 30 and the other one half of his assignment is devoted to administrative duties. His background includes six mathematics courses and two methods courses in mathematics. Bill rated the adequacy of his background for teaching electives, mathematical problem solving, mathematical applications and implementing the overall philosophy, goals and objectives of Math 30 as "very good". However, he rated his statistics background as "good" only.

Bill's School Setting

School B, in which Bill teaches, has a total enrollment of approximately 225 students and it offers programs from grade eight to twelve. There are 15.5 full-time equivalent teachers on staff with five being involved in teaching mathematics. The school is only fifty percent utilized with twenty percent of the students residing in the community in which the school is located and eighty percent of the students reside in neighboring communities and the surrounding area. The school operates on a semester basis at the high school level. Five broad programs including an academic program, business education, fine arts (band and drama), home economics/industrial arts and work experience are offered at this school.

The community has approximately 700 residents, many of whom are retired from the surrounding farms. Many residents in the area are self-employed farmers and the service industry, provincial/federal agencies, local business and the school system are major employers in the area. A variety of recreational facilities and a public library are available for the residents.

There are a number of school setting factors that have an influence on

Bill's interpretation of curriculum materials. While there are, no doubt, many factors of influence, only those that repeatedly surfaced are described in this section.

Bill is quite satisfied with the physical setting in which mathematics is taught. However, there are other school setting factors that emerged as strong influences on Bill and his interpretation of curriculum materials. Probably the most powerful factor is the student text, which he describes as frustrating to the students and himself. It contains many mistakes in the body of the text, the examples and the solution section. Also many sections provide totally inadequate coverage of particular topics. In addition, he describes it as a poor resource for mathematical problem solving. As a result, Bill has no choice but to draw a considerable amount of the content used from supplementary resources and prepare his own material. The issue of the textbook surfaced repeatedly and emphatically throughout the study. Bill views the text as an important resource for the students and for the content of his lessons. He suggests that this text could be devastating to a beginning mathematics teacher. At least he sees himself in a position of having the confidence, background and expertise to reassure students that their solution is correct and the one shown in the text is not. Another concern with the text is that on some occasions Bill would cover a topic only to the extent that the text did and not to the extent required by the curriculum. Conversely, content is taught to students because it is in the text and not necessarily required by the curriculum.

The size factor, as it relates to the school, staff and class, also emerged as a strong influence on Bill's classroom practices. With respect to the school size, Bill feels that in a small school, students are often drawn into Math 30 because of limited course offerings. This has the tendency to widen the ability range of students in this course significantly, presenting special challenges for the teacher. Bill feels that under these circumstances he can no longer teach at the "Math 30 level", but must now find appropriate strategies to teach students below the "Math 30 level".

The number of mathematics teachers and the level of interaction among

them is also seen as an influence on his classroom practices. Although there are five teachers involved in teaching mathematics, only one is a qualified mathematics teacher who does not teach the same courses. As a result there is little or no interaction on curriculum and classroom related matters. Bill considers the absence of such professional interaction as negative, leaving him sometimes wondering if he is doing what the curriculum intends him to do. The factor related to class size has in this case a real positive influence on Bill's classroom practices. He remembers from other years when his classes had in excess of 25 students enrolled, his ability to address individual interests and abilities was greatly diminished. The current class size of thirteen students allows him to use core extensions and different teaching strategies far more readily. He feels he can truly reach all students and address their needs.

Bill continues to teach graphing by drawing the graphs on the blackboard instead of using computers. Although he continues to point out that his lack of skill and background with computers prevent him from using them in his class, he has now convinced himself that his "blackboard and chalk" approach is probably just as effective and probably less time consuming. This is so incongruent with Bill's otherwise adventuresome attitude in the area of problem solving. Calculators, on the other hand, are not only encouraged in Bill's class, but students are required to have scientific calculators during his mathematics classes. He sees and understands the potential of calculators in his classroom, but he seems to have rejected at this point the even greater potential of computers for his classroom.

The expectation of the community are also felt by Bill as a factor that influences his classroom practices. He seems particularly concerned about the public perception of student failures on the MDE. His frequent reference to the community and the MDE prompted the question, "Why are you so worried about the diploma exam?" Bill responded:

"I am not really worried, but the achievement on the diploma is pretty public. The public doesn't understand that some students should not have been in Math 30, but their failure is directly related to the quality of my teaching. Of course, I work as hard as I can with weak students and I provide all the help possible, but that is sometimes not enough."

Bill feels that the community may not recognize other factors related to such failures. The influence of this community factor has caused Bill to re-examine his approach which has resulted in a shift from being process oriented to being somewhat product oriented. His classroom practices now include a greater emphasis on reviewing and writing previously written MDE, teaching more toward the MDE, limiting the content more consciously to the core program and excluding elective topics.

Bill's View of School Mathematics

Bill's predominant view of school mathematics is one of a problem-solving activity with exploration being an important element. In terms of content, mathematics for him is a complex body of knowledge comprised of axioms, propositions and concepts that are logically connected. He feels that this body of knowledge and the logical connection of its parts must be explored by students if they are to develop an image of mathematics that is one of "common sense and not a mystery". School mathematics, for Bill, is common sense and he wants students to have this image of it. He describes it as "... true to nature and it's not a mystery or anything else". His view is that one "... gets students to learn it by giving them basic understanding and not through memorization of facts and data". Bill feels that mastering the basic skills and understanding the logical connections within the structure of mathematics are essential to develop a view of mathematics as a unified whole. In order to achieve that view of mathematics, Bill injects frequent reviews in his lessons which focus on previously learned materials. When asked about these reviews, he replied:

"You simply can't take it for granted that students have retained all the previously learned facts and information. So, a quick review is helpful. But, I also think it is important, because I show them the relationship between things learned today with those learned before... and I think it helps students to better understand the 'big' picture."

Further to that, Bill spends a considerable amount of time developing formulas, theory, and doing proofs in the hope that students will grasp the reasons behind these formulas. He subscribes to the belief that if students have a good grasp of basic skills and a good understanding of mathematics, they will see applications for it. Among his other justifications are that students really benefit from seeing such proofs demonstrated or doing some themselves. He added that many students find it interesting and they enjoy the thinking process and the challenge. For Bill, mathematics is synonymous with problem solving and exploration and he sees that having a positive impact on the students' non-mathematical problem-solving skills as well. He often engages with the students in discussions on the transfer of mathematical problem-solving skills to problems outside the mathematics classroom.

While Bill clearly values the nature of mathematics and the theoretical concepts and principles inherent in the subject, he adheres, for the most part, closely to the core curriculum. The absence of elective topics is indicative of that. In general, Bill is very conscious of the core curriculum objectives and their appropriate coverage. He is satisfied with the content of the trigonometry unit and feels that there is no need to extend the unit. However, on occasion he does extend core topics in areas such as trigonometric formulas, proving trigonometric identities and mathematical applications. Extensions of core topics occur in Bill's classroom primarily as a result of his own beliefs and values about mathematics. However, they also arise quite often out of a need to respond to specific student interests. They always seem to happen spontaneously and never on a pre-planned basis. For example, Bill believes that proofs unify basic ideas in mathematics and that students need some exposure to the nature of formal mathematical proof. Although he recognizes

that students find the derivation of formulas and doing proofs the most difficult, Bill continues to expect these from his students, even though it is not required in the curriculum. Guided by that belief he provides a formal proof for nineteen of the twenty-seven formulas listed on CPTT survey. The negative arc formulas for sine and tangent and the complementary arc formulas for sine and cosine are simply stated and an informal justification is provided. Half angle formulas for sine, cosine and tangent and the double angle formula for tangent are not covered at all. Bill's depth and breadth of coverage of core content exceeds considerably the expectations of the curriculum. These extensions of the core curriculum seem to be Bill's way of dealing with the elective portion of the program. This is also evident with respect to the use of various trigonometric interpretations. While the curriculum includes the use of the ratio interpretation and unit reference circle, Bill covers the line representation of trigonometric functions and the wrapping function interpretation as well. His justification for this extension is as follows:

"I favor using several interpretations because some students may just favor one over the other. Of course, trig ratios in right triangles I emphasize a lot as well as the unit circle. The old course emphasized the wrapping function development, so I keep on using it as well because some students find it easy to understand. The line representation of trig functions is also used a fair amount in my class. I think aside from one particular interpretation being easier for some students than the other, I think they get a real good appreciation of their relatedness and consequently a better understanding of trig."

These comments reveal much about Bill's thinking. They show that he is concerned about the students' understanding, that he still has not let go of the previous curriculum, but most important, that he is anxious that students grasp the structure of mathematics and its parts. There is no doubt that Bill views trigonometry as the most important unit. This is evident by the amount of time spent in this unit, the evaluation plan and the number of supplementary resources used. When asked what he considers to be the most important concept in this unit, he responded:

"... I guess, the concept of the unit circle. Of course, that means a lot. It carries on a lot, but I think if I can get them to understand the concept of the unit circle and the radian measure there, then the rest comes. I think they understand the rest if you can give them that basic bit across and the related angle concept in the unit circle."

The concept of amplitude, periodicity and phase shift do not receive the treatment that Bill feels they deserve. He suggests that this is partly due to his inadequate background, but it is also due to the inadequate treatment of these concepts by the text. They seem to be treated like "add ons and left, and may be that's why I teach it that way."

The various sources of evidence suggest that Bill holds strong values and beliefs about school mathematics which include valuing mathematics as a unified whole, valuing mathematics as a problem-solving tool, valuing the nature of mathematics and valuing theoretical concepts and principles. For Bill, knowing mathematics means understanding the structure of mathematics and the logical connection of its parts. Although this view of mathematics guides all of Bill's classroom practices, he finds that he has to compromise his values and beliefs from time to time in order to prepare his students for the MDE. While the MDE has a powerful influence on Bill, it is not the only influence on his classroom practices. Bill's selection of curriculum materials is influenced by other factors such as: Math 30 Curriculum Guide, what students require in order to understand the structure of mathematics, approaches that have worked well for me in the past, what students will need to know in order to succeed in their post-secondary education, what students will need to know in order to understand the importance of mathematics in their daily life, what society needs by way of informed citizens, what students find easy to understand, the students' ability and the time available. Other factors had little or no influence on Bill's interpretation of curriculum materials for use in his classroom.

Bill's Teaching of Mathematics

Bill's teaching follows a well established routine consisting of: reviewing previously taught material, solving problems that students had difficulties with, introducing a new topic, checking student understanding of newly taught material, assigning student practice exercises, assisting individual students in class and assigning homework. Such routines as taking attendance is done at the end of the period or when students work on their own. On days when there is no homework, Bill briefly reviews the highlights of the previous lesson and leads into a new lesson for the day. It is not uncommon for Bill to begin a new lesson within a problem-solving context. This involves a problem that contains some or all of the concepts to be taught. For example, he began the introduction of the law of cosine by posing a practical problem involving an oblique triangle. Students quickly realized that their current knowledge was inadequate and that some other tools were needed to solve this problem. There was much discussion among the students. Students became increasingly more anxious to solve this problem. Bill eventually proceeded with the proof of the law of cosine, but he constantly connected each step with the problem that was posed. In this discussion he also incorporated a review of the Pythagorean theorem, the naming of triangles by sides and angles, and the limitations of the trigonometric ratios. As Bill and the students embark upon a solution process, it is common practice for him that the concepts to be learned are highlighted, explained and applied within the context of that problem. The students seem to enjoy this approach and Bill feels that it is effective.

On days when students had to complete a homework assignment, Bill selects several students to write out their solution steps to a particular question on the blackboard. While this is being done, Bill moves around the classroom interacting with the other students. Once the students are finished at the blackboard, Bill checks each solution quickly, placing a check mark on those that are correct. Students who have made a mistake are asked to correct it. The students at their desks are expected to check their homework against the

solutions on the blackboards and they are encouraged to raise questions about these solutions. During this process, there is much interaction between Bill and the students and among the students. Bill values the contributions made by students and he is quick to recognize teachable moments particularly when they are created by students. Among the many values he holds on teaching, the most prevalent is his emphasis on teaching mathematics as a process. He enjoys exploring mathematical problems and questions with his students and he is constantly connecting the newly taught material to the "big picture." When he demonstrates or discusses a problem, he keeps on posing new problems, suggesting alternative solution methods and encouraging students to ask questions. His approach is similar to the Socratic method of interacting with students. From the classroom observations it is evident that Bill likes to "think out aloud", ask questions, wonder where this or that solution path may lead in short, he simply enjoys "losing himself" with his students in a mathematical problem. Bill is not afraid to pursue an unknown solution path and he often prefaces his problem solving by saying, "let's see what happens if...". His problem solving approach and his teaching style are synonymous and it occurs naturally. As a result there is a high degree of problem solving observed with all four indicators - understanding the problem, devising a plan, carrying out the plan and looking back - being present. These problem-solving steps are not applied in a mechanical way, but rather through questions like: "Well, what do we know about this problem?" or "Can you think of other methods?" or "How do we know the solution is correct?" or "Can someone think of a shortcut or a different solution method?" Bill feels that his emphasis on teaching mathematics as a process is essential if students are to gain an understanding of the theoretical concepts and principles that pervade mathematics. Bill emphasizes consistently understanding in contrast to memorization of facts. He wants students to understand where the formula is derived from, why it is there and what value it has in the context of that topic, instead of saying, "here it is and plug in numbers." However, he recognizes also that this orientation requires considerably more time than a lecture or drill and practice approach.

Bill is quick to point out that he had a real conflict between preparing his students for the MDE and preparing them for further study at university. The former, because of its very nature tends to focus exclusively on preparing students for that examination, while the latter demands a process orientation with an emphasis on exploration and problem solving. Bill is concerned that an emphasis on preparing students for the MDE is done at the expense of preparing students for further study at post-secondary institutions and vice versa. He states:

"I think by preparing them...I prepare them for the examination and not prepare them as well for university or vice versa. I don't have time maybe to do both. For instance, the derivation of formulas and that kind of thing which has so far never been tested, I think is a very integral part of the course and since it is not on the diploma exams you have a tendency over a period of time maybe to slack off on those and not make those expectations of the students the same and yet for university, I think that is a key part."

Bill identifies the diploma exam as the most critical factor affecting his interpretation of curriculum materials and subsequently his teaching. The following comment alludes to that:

"...and I don't like to think that it is structuring me, but I feel it is and it is modifying my approach all the time. A little bit every year, because you want the students to do the best they can on their examinations, because it is very important for them and again, I mentioned, I find that conflicting with me actually teaching them how to think and reason and prepare for the university type situations in mathematics."

Bill's evaluation of students is completely focused on measuring the students' knowledge and understanding of mathematics and how they apply it to problems that confront them. His concern with their understanding of the core content is particularly prevalent because the MDE is comprised of that content only. Bill's preoccupation with this factor is particularly evident from his frequent remarks about the MDE to students in class. He is also conscious of the fact

that the MDE focuses on the product - that is, on the correct solution - and not on the process. This realization has caused Bill to change his teaching approach from a predominantly process-oriented style to in emphasis on the product. His struggle is expressed in the following comment:

"... as much as I fight it, the diploma exam changes my teaching every year. On the one hand I want to teach them mathematics and provide them with a solid base for study at university and, on the other hand, I have to prepare them for the diploma exam. These two things are, in my opinion, not the same. So, I strike the happy medium between the two, but I still think I do it at the expense of teaching them mathematics."

Although Bill feels at times that the MDE is a serious threat to his process orientation, he continues to display it as a prominent characteristic of his classroom practice. However, he is now more conscious of his time and after several lessons using a process orientation, he returns to a more formalized classroom routine. This routine generally follows the pattern of reviewing, answering questions, lesson development, checking student understanding, student practice and assigning homework. Bill describes this routine as being pedagogically more efficient in terms of covering content. Pedagogical efficiency, however, is not just evident in his choice of teaching strategies, but also in a number of other areas. Limiting the program to the core curriculum and omitting core curriculum objectives is done because of a time concern. The decision to administer frequent quizzes instead of tests is because of time. He limits problem solving of real-life application because of time. He can no longer "loose himself" as often and as long with students in good mathematical problems because he falls behind in other areas. He finds that his time is limited to include content that is interesting or has arisen out of a particular teaching moment. Bill relates his time concerns directly to the MDE, when he states: "Before we had diploma exams, I used to pursue some of these interesting questions and problems, but now you have to get your kids ready for the exams." Closely connected with this problem is his inability to modify the

content for students who experience difficulties with it, because if he would these students would not be properly prepared to write the MDE.

Another value, closely related to pedagogical efficiency, is academic rigor. This value is particularly evident in the area of content. Bill injects rigor in his classroom by extending the core topics, requiring proofs of several trigonometric identities and the law of sine, and including various trigonometric interpretations. In addition, the textual content is being supplemented with numerous teacher-prepared worksheets and materials. Another indication is his expectation relative to homework. He indicates that: "At this level I expect students to do homework. My philosophy is that I will not assign work just to keep students busy, therefore, when I assign something I consider it to be important and I expect it to be completed. Even when students miss my class, that does not give them the excuse to miss the assignment."

Another prominent characteristic of his teaching is his valuing of student understanding and success on the MDE. There is frequent reference to the MDE and his use of previously written MDE for practice purposes is further evidence of that. There is no doubt that Bill's emphasis in deriving formulas and doing proofs stems partly from his desire to increase student understanding.

Motivating students is another strongly held value by Bill. He achieves this through his wit and tremendous sense of humor. Students seem to enjoy his classes and the good natured bantering that goes on. He also succeeds in motivating the students through his particular teaching approach that invites active student participation and questions, examines problems from different perspectives, values and appreciates student contributions and provides a secure and non-threatening environment for students. In addition, he selects interesting problems in response to particular student needs. The following is in response to the question, why he often asks his students for alternative methods of solving a problem. The response illustrates the kind of atmosphere within which his students work:

"Habit, I guess ... (laughter). I have actually several reasons. Kids often have interesting solutions that I have not even thought about or some of the solutions may be easier for some kids to understand. Of course, I also think that the more kids solve problems using a variety of solution paths, the better problem solvers they will become."

Despite Bill's relaxed teaching style and the informal atmosphere in his classroom, there is a general sense of a productive environment. Bill is also willing to assist students at any time if they experience difficulties. All of these gestures seem to have a positive effect on the students' motivational level. However, Bill is not prepared to compromise his high and clear expectations of students. Students receive at the beginning of the program a course outline that is very specific in terms of expectations relative to content objectives, evaluation, class work, homework, attendance and a few other matters. Among his various expectations are that students work cooperatively and productively in the classroom and that students are expected to do homework, even if they are absent on a particular day. Bill also expects students to have calculators so they can focus on the higher thinking skills rather than doing basic arithmetic, as he puts it. His expectations for mastery of content are high and quite clear. Students are expected to master even the extended content, like proving trigonometric identities and the law of sine. It appears that the students are responding positively to these expectations.

The value that Bill places on feedback on student performance and professional competence are two characteristics clearly evident in his classroom practice. The first is illustrated by Bill's comments when he says: "I do find that quizzes give me very quick and important feedback on how well students understand. They take very little time, provide students with much needed practice and they represent a controlled check on individual performance." This desire for constant feedback on student performance is further illustrated by his teaching approach. In his ongoing dialogue with students he is constantly checking verbally for student understanding. Also his entire evaluation scheme is focused on measuring the students' knowledge and

understanding of mathematics. It appears that Bill wants to be in a position to know the students' level of understanding at all times, so that any perceived problems can be addressed immediately.

The second characteristic, valuing professional competence, is evident in one area related to the use of computers in his class. He is clearly reluctant to use computers to teach graphing because of his lack of background and knowledge about computers. In the interview he stated that he wants to achieve a certain level of competence with computers first before he will use them in his class. This is somewhat incongruent with his venturesome attitude in problem solving where he is prepared to "lose himself" with the students to solve problems.

Similar to Alice, Bill tends to return to approaches that have worked for him in the past and for which he has a strong background. He is reluctant to venture into areas for which he feels inadequately prepared. The use of computers in his classroom and the treatment of the concepts of amplitude, periodicity and phase shift are two such examples. Bill states that computers are not used at all and he admits that the concepts of amplitude, periodicity and phase shift receive only superficial treatment. As a reason he cites his own mathematics education which did not encompass a thorough treatment of these concepts.

Bill's Students

Bill and his thirteen students enjoy a friendly relationship with each other. The students seem to enjoy Bill's humor and he enjoys having fun with the students. He encourages the students and they are comfortable asking questions. Bill's relationship with his students is characterized by his focus on three areas: students' ability, students' interests and students' contributions. His awareness of the students' ability is evident from his comment: "... I know the students since grade eight and by the time they reach grade twelve I have a good handle on their mathematics ability." Bill considers himself quite fortunate

that his class is small, because this allows him to get to know the students even better. Based on his extensive personal interaction with each student and the wide variety of student evaluation activities, Bill seems to know exactly what each student can do. If any of his students experience difficulties in his class, he encourages them to come for extra help. During his lessons it is obvious that Bill is very cognizant of the fact that some students are experiencing difficulties. He would then search for alternative explanations, engage in a discussion with the student suspected of having difficulties or provide one-on-one assistance later in the lesson. It is not uncommon for other students to come to the rescue and assist that student who experiences difficulties. Bill recognizes that some students simply need more time, extra practice and modified explanation. He does not think that he could or should modify the content to meet a students' needs, because the MDE demands a certain standard that all students are expected to meet. The difficulties that some students are experiencing in mathematics are attributed by Bill to their inappropriate placement. Instead of these students being in Math 30, which is the academic course for university-bound students, they should have been placed in Math 33. Bill thinks that this student placement problem is typical for small schools in that a few students always end up being in Math 30 for a variety of reasons. Inappropriate placement is not the only reason for experiencing difficulties. According to Bill, some students simply do not apply themselves as much as they should and others are affected by factors originating outside the school. Despite the fact that some students are inappropriately placed, Bill still prefers having them in Math 30 as long as students are making an effort to succeed. He feels if students have the right attitude and with his willingness to help them inside or outside of class, they have a good chance of succeeding in Math 30. While Bill recognizes the wide range of interests and abilities in his class, he operates on the assumption that his students have a good attitude toward learning mathematics and a good understanding of the content. However, he does not simply take it for granted, but rather, he works toward those ends consciously. Based on the results from the ATMT, Bill's effort produces results. With respect

to Mathematics in School, the students' responses show that all nine stated activities are either "very important" or "important". In particular, students are saying that, checking an answer to a problem by going over it, is very important. This is consistent with classroom observations which confirm that this activity is more than providing students with the correct answer or confirming it. It incorporates elements of review, teaching, remediation and extension. Solving trigonometric application problems is also viewed as very important by the students. This activity reflects Bill's continued emphasis on problem solving using real-life applications. The overall findings on this attitude dimension suggest that students feel positive towards Mathematics in School and they like it. However, the importance that students attach to Mathematics in School is rated higher than the degree to which they like it. With respect to the students' view of Mathematics as a Process, the responses suggest that students view trigonometry as a set of rules. These rules are used in solving problems and, in fact, the problems cannot be solved without the use of these rules. This view about trigonometry seems to have been shaped by Bill's emphasis on developing formulas and theory and exploring the reason behind these. However, his students also feel that trigonometry helps one to think logically, that it is a good unit of study for creative people and that trigonometric problems can always be solved in different ways. Again, these attitudes seem to be reflective of Bill's classroom practices. The dimension, Mathematics and the Student, which measured the students' personal feelings toward mathematics suggests that students want to do well in trigonometry, they feel good when solving a mathematics problem by themselves, they feel challenged when given a difficult mathematics problem and they think mathematics is fun. The overall findings in the ATMT suggest that Bill's students have indeed a positive attitude toward mathematics.

Bill's assumption that his students have a good understanding of the trigonometry unit is not strongly confirmed by the results of the MDE. His students achieved slightly below (62%) the provincial average (64%) for trigonometry alone and also slightly below (62%) the provincial average (63%)

on the overall Math 30 achievement. He shows considerable sensitivity and responsiveness to individual abilities in his planning and through his classroom strategies. His focus on the students' interest is evident as well from his classroom practices which include primarily spontaneous extensions of core topics. Bill's planning and the classroom observations suggest that these core topic extensions are not planned in advance. The following comment supports this observation:

"... if there are certain areas where the students are interested in then I'll branch off and maybe get caught up for a few days in graphing in general or something of that nature instead of going into a specific one."

Bill is suggesting that if students show an interest in graphing, he would willingly extend this topic beyond the graphing of trigonometric functions and explore with students the behavior of other functions. This "branching off" varies from year to year depending on the students, their abilities and interests.

The value that Bill attaches to students' contribution is a strong characteristic of his classroom practice. He constantly engages students in the discussions, encourages them to make suggestions and even counts on them to produce different solutions to the same problem. Bill says: "... kids often have interesting solutions that I have not even thought about ... ". The "blackboard drill", which involves students showing their solution steps to a particular problem on the blackboard, is considered by Bill as an opportunity to engage students in a "rich" discussion on alternative solutions. The students feel that this is an effective strategy for learning mathematics and they appear to be comfortable with the "blackboard drill".

Bill's classroom organization seems to keep all students productively involved while at the same time creating a cooperative learning environment. The students seem to be on task for the duration of the entire class period. Aside from Bill having expectations that students are in his class to learn mathematics, he also selects teaching strategies that ensure that his students

are productively engaged. Among these are teacher demonstrations, drill and practice and checking homework by having students solve the problems on the blackboard.

All the sources of evidence suggest that Bill's selection of curriculum materials for use in his classroom is strongly influenced by student-related factors. These include the students' ability, what students will need to know in order to succeed in their post-secondary education, what students will need to know in order to understand the importance of mathematics, what students will need to know in order to do well on the MDE, what students require in order to understand the structure of mathematics, what students find easy to understand and what students will enjoy and find personally interesting.

Summary Statement of Bill's Case

Bill is a confident mathematics teacher at the level of high school mathematics, who has a strong process orientation in his teaching. His emphasis on problem solving and mathematical explorations is contained in his view of school mathematics and in his teaching. His curiosity in the subject and his confidence in problem solving provide the students with many opportunities to discover new mathematical ideas and relationships. Bill feels that mastering the basic skills and understanding the logical connections within the structure of mathematics are essential to develop a view of mathematics as a unified whole. Bill's overriding goal is for students to be at ease with mathematics, seeing it as "common sense and not as a mystery."

The existence of the MDE has created a real conflict within Bill's pedagogical perspective. The conflict centers around the question of whether or not he prepares the students for the MDE or for further study at university. In Bill's opinion, the former implies teaching with a product orientation and the latter with a process orientation. The result is that he strikes a happy medium between the two, but, in his opinion, at the expense of teaching mathematics.

Bill has not yet accepted the computer as a tool to enhance learning in

the mathematics classroom. His lack of skill and background with the computer are used to deny its relevance in his class, implying that he is justified to avoid them. This position is completely incongruent with Bill's mathematical disposition toward problem solving. Bill's reticence about using computers continues and so does the obstacle that stands in the way of challenging his students even more through the use of computers.

Bill shows considerable awareness of the many influencing factors on his view of school setting, school mathematics, teaching and students.

Carl - the rule developer

Carl is in his early fifties, has thirty years teaching experience and twenty years experience teaching high school mathematics. Although his specialization is mathematics, which includes seven mathematics and two methods courses, he is currently teaching seven different courses of which only two are in mathematics. Carl teaches 124 different students from grades seven to twelve daily. He rated the adequacy of his background for teaching electives, mathematical problem solving, mathematical applications and for implementing the overall philosophy, goals and objectives of Math 30 as "very good." However, his statistics background was rated by him as "unsatisfactory", whereas the physical setting in which he teaches mathematics was rated as "good".

Carl's School Setting

Carl teaches in a kindergarten to grade twelve school which has a total enrollment of approximately 325 students. Of the staff of nineteen teachers, nine are involved in teaching arithmetic/mathematics. However, only three of these nine teachers teach at the junior-senior high level with Carl being the only teacher offering Math 30. The school offers all three streams of mathematics, but low enrollments at the high school level require that the grade 10 and 11

level courses for two streams be combined. Math 30 is offered and taught on its own. The school is approximately eighty percent utilized with nearly all students being bused. Approximately three percent of the students reside within walking distance around the school. School C operates on a ten-month program at all levels. It offers four broad programs which include: academic, business education, fine arts courses (junior/senior band and drama) and work experience.

School C is a partly single and partly two storey wood frame construction building which was built in numerous phases since 1952. In 1985, a major renovation and modernization was undertaken of the entire structure.

There are approximately fifty residents living in the hamlet surrounding the school. Some residents are self-employed in local business, some travel to the larger community for employment and others are employed in the local gas and oil industry. Residents from the surrounding area are primarily farming and some are living on acreages. The school houses the public library, a swimming pool is located adjacent to the school and an open hockey rink and community hall are located across the street. All these facilities are available for school and public use.

The influence of the school setting on Carl's interpretation of curriculum materials emerged as a strong factor. However, this factor has several elements that relate to the organization, people and resources. The organizational elements include Carl's teaching assignment and school organization.

Carl's teaching assignment is a recurring theme which affects him in many different ways. With respect to the use of resources, Carl states:

"...I teach seven different courses from junior high to senior high and they are not all mathematics. I simply haven't got the time to look at or use too many different resources. It takes all my time and energy just to offer the basic program in each. I have to forget the fancy stuff."

In response to his teaching he states:

"As I indicated to you before, my teaching schedule is such that I find little time to reflect on my lesson."

The influence of this factor does not seem to end here. Carl claims that this factor seriously limits his ability to plan, evaluate and put effective classroom practices into place.

Closely connected with the teaching assignment is the school organization. The ten-month program, as opposed to the semestered program, has some serious disadvantages according to Carl. This is particularly the case in a school like School C, which houses grades K to 12 and is relatively small. The nature of the teachers' assignments and the number and length of class periods becomes unsuitable for many subject areas.

The people element of the school setting factor relates to the number of staff members, the nature of the students in Math 30 and community expectations. Carl expressed concern with being the only high school mathematics teacher on staff. He states:

"You want to talk about the curriculum, teaching or a particular assignment with another teacher-just to see if you are on the right track-but here is no one to talk to. Sometimes, I am not sure that I am doing the right thing."

Carl suggests that many aspects of his program would probably change if he had the opportunity to discuss these with another mathematics teacher teaching the same course.

With regard to the nature of his students, Carl feels that some students are enrolled in Math 30 because his school is small and alternative programs are not available for these students. Once these students are in his program, there seems to be an onus on the teacher to make these students succeed. In many cases success is impossible and in some it requires substantial remediation plans.

The expectations of the community are an important influence on Carl's classroom practices. He feels the community would not accept the failure of too many students on the MDE, therefore he is constantly searching for more effective ways to make his students succeed on this exam. Carl suggests that a high number of student failures would reflect on his effectiveness as a teacher.

The third element of the school setting factor relates to the resources. In particular, the textbook emerged as a powerful influence with the computer as a distant second. The text is a constant source of frustration for Carl. He describes it as too teacher-dependent, poorly presented, not sequentially organized, full of mistakes and incomplete in some content areas. Carl sees it as a poor match between the mandated curriculum and the prescribed text. He indicates that a good text is even more important in a setting such as his, because much of what he is doing is guided by it. Carl admits his discomfort and dependence on the text.

"...I feel very uncomfortable with this text. It contains many mistakes and it includes content that no text before dealt with (eg. volume of sphere). It isn't very sequential either. I depend a lot on the text. This is my primary resource. In fact, nearly my only resource I use and if it isn't O.K., then I get worried. You see, I rely on the text because I teach seven different courses from junior high to senior high and they are not all mathematics. I simply haven't got the time to look at and use too many different resources. It takes all my time and energy just to offer the basic program in each."

Carl clearly limits his course to one textual resource with occasional reference to an obsolete text from the previous curriculum. However, he does not really want to use additional textual resources because:

"My students are more comfortable with the use of one text and actually so am I. Using more than one text is often confusing because of the different terminology and treatment of particular topics. That's why it is so frustrating when the authorized text is a poor one."

The use of computers is for Carl essentially a non-issue. Although he talks about computers and sometimes appears to worry that his students are not exposed to them, however, he is not prepared to use them because he lacks the skill and background to use them. Carl has a classroom set of scientific calculators and makes these available to the students during the mathematics classes.

Carl's View of School Mathematics

Carl views school mathematics as a set of topics each with a body of content that contains skills, rules, procedures and principles to be memorized and applied when solving problems. In Carl's class, the treatment of these topics often occurs in isolation, making only occasional reference to previously taught material. There is strong agreement across the sources of evidence that Carl's view of school mathematics is shaped by his belief in the importance of students being able to recall facts, produce the correct solution and master basic skills. A skill-oriented view of school mathematics, with an emphasis on mechanical procedures to be memorized and applied is clearly practiced in Carl's classroom. This emphasis on recalling facts is also evident on his quizzes which often contain several exercises of the same kind. On another occasion he would practice for an entire class period the use of trigonometric tables and finding trigonometric values using the calculator. The following comment by Carl seems to reveal the importance of that practice to him:

"Generally speaking, I lean more in the direction of teaching students basic skills, like being comfortable with using trig in solving right triangles and applying trig to application problems. I also want them to have good skills in using the calculator because that is most likely the device that they will use in solving any problems."

This view of school mathematics is further illustrated by his comment that the essential pieces of knowledge and facts that he expects students to

understand and remember are the three primary trig ratios, the reciprocal trig ratios and the coordinates of the points on the unit circle. Carl's emphasis on teaching basic skills and the "little parts" as he calls them is clearly stated in the following:

"I think for them to grasp the whole of trigonometry...all of these little parts, if you want to call it that, are important to the student. It's all part and parcel of the whole."

Carl feels that unless students understand the "pieces", they will not understand the "whole." This reasoning gives him the license to emphasize his skill-oriented approach. Even the nature of his tests reflects an emphasis on the recall of facts with a focus on obtaining the correct solution. His rationale for that is best described in the following comment:

"There is no doubt that the bottom line is getting the correct solution. It doesn't matter if an engineer shows a nice solution path and the answer is wrong. I know that this isn't quite the same, but even the diploma exam focuses, for the most part, on getting the correct solution."

Carl's theory-in-use seems to subscribe to the more "practical side" of trigonometry rather than the "theoretical side". He feels that the abstract concepts (or theoretical side) of this unit should be downplayed or even excluded. Carl refers particularly to the derivation of trigonometric identities and non-application related material. His preference is clearly what he calls the "practical side" of trigonometry. These include such activities as: maintain previously developed skills, trigonometric equations, identities (confined to quotient, reciprocal and Pythagorean), solving oblique triangles using the sine and cosine law, determine coordinates of points on unit circle, define trigonometric ratios in terms of coordinates of points on unit circle, evaluate five trigonometric ratios given the value of one, determine exact values of trigonometric ratios of 0° , 30° , 45° , 60° and 90° and applying sine and cosine law to practical problems. The "theoretical side" of trigonometry, such as

derivations of complex identities or relationships and the concepts of amplitude, periodicity and phase shift, receives very minimal attention by Carl. The practical side of trigonometry and mastering basic skills seems to be synonymous for Carl. In fact, mastering basic skills is one of his strongly held beliefs and it is consistent with his view of mathematics and his actual classroom practice. The importance of this belief to Carl finds expression in the many reviews that occur in his classroom. On one occasion he reviewed material with three students because of their absence the previous day, while the other seven students did exercises from the text for the entire period. However, his reviews of definitions, rules and procedures are mostly done in isolation from the "here and now" and they are seldom connected with previously learned concepts. Even when students ask questions beyond the core objectives, Carl normally limits the discussion to a level not outside the core curriculum. On one occasion a student asked how the CAST rule relates to the reciprocal trig ratios, to which Carl responded by explaining the sine, cosine and tangent functions again. When asked why he avoided the student's question, he said: "Because I don't think that they understood the CAST rule in relation to the sine, cosine and tangent functions. And, had I added the cosecant, secant and cotangent function to this discussion I would have confused most of them. Sometimes it is better to understand one thing well instead of two things poorly." This kind of student-teacher exchange is not uncommon in Carl's class.

This view, that understanding one thing well is better than understanding many things poorly, is again expressed by Carl relative to considering alternative methods of solving problems. He states his view as follows:

"...I have always preferred that students understand and can use one solution method properly than none at all. I found that too many different ways of doing things confuses students instead of it clarifying the problem for them. I will certainly accept alternative methods from students as long as they are comfortable with their chosen method. However, in my teaching I would not say, here are the five methods of solving the problem."

Keeping the mathematical content within a well defined area is closely connected with Carl's other belief that his program consists of the core curriculum. His strict adherence to the core curriculum is made quite clear when he says that interpretations of trigonometry other than the ratio and unit circle interpretation are not covered in his course. These two are in the curriculum guide, while the other interpretations are not. He adds: "...the core curriculum contains plenty of stuff and if you want to cover it well so that students understand it and are ready for the exam, the 125 hours is barely enough." When Carl was reminded that the 125 hours also include the elective component, he stated:

"I hate to admit it, but I don't have an elective component as such in my course. On occasion I extend a core topic a bit when I think students find it interesting or it challenges some of the better students, but generally speaking, my program is the core program..."

In actual practice, extensions are not occurring in Carl's classroom, unless he is referring to a unit other than trigonometry. However, in another interview he indicated that he is not extending core topics in other units either. His comment is:

"No, not really. I need all the time, including the time available for electives to cover the core program. Also, the time required to prepare for such extensions is simply not available. Aside from the time problem, I also feel that the students have plenty to chew on just by focusing on the core program."

Carl's treatment of the trigonometry unit is not only limited to the core curriculum but it is also reduced in some areas. Some curriculum objectives are not covered or they are limited to the coverage in the student text. In Carl's class, the student text is the program. For example, several trigonometric identities are not derived and students are merely expected to recall and apply them. The findings suggest that Carl adheres closely to the core curriculum with an emphasis on those objectives that are assumed to be on the MDE. Although the Math 30 Curriculum Guide is his primary source when deciding on the inclusion or exclusion of content, the MDE is certainly the second most important reference when making such decisions. The exclusion of the wrapping function and line interpretation are a case of point, which were excluded because they were not assumed to be on the MDE. The various sources of evidence suggest that there are two important factors that influence Carl's selection of curriculum materials for his classroom: "What students will need to know in order to succeed in their post-secondary education" and "What students will need to know in order to do well on the MDE." Carl confirmed the strong influence of the MDE on his selection of curriculum materials on several other occasions, suggesting that the MDE is primarily responsible for limiting the program to the core curriculum and his emphasis on skill development.

Carl holds two other strong values about mathematics: as a powerful tool to exercise students' minds and teach them to reason and as a tool for solving mathematical applications. With respect to the first value he states:

"...the thing that is so important to me, so vital and which I find lacking in people in general in the world...and that's one of the things I think mathematics is so important for and that is developing the ability to reason, to think out what has happened, why it has happened, to grasp or to see through to the end, what the consequences of things are and this ability to reason...to think is something that has to be developed and I think that math is the tool that is necessary to do this."

There is no evidence that this value is reflected in Carl's classroom practice. This statement belongs, at best, in the realm of Carl's espoused

theory, suggesting that he is searching for an orthodoxy which is not yet practiced in his classroom.

His claim that he values major mathematical concepts and principles as they relate to mathematical applications is also strongly stated with respect to his treatment of the concepts of periodicity, amplitude and phase shift. However, when examined in the context of real classroom practice, Carl gives these concepts superficial treatment.

The overall findings suggest that Carl's view of mathematics is that the program consists of a limited core program which operates below the expectations of the curriculum.

Carl's Teaching of Mathematics

Carl's teaching practice follows a well established routine that always begins with responding to questions from the previous lesson. Invariably, Carl uses about one half of the class period to respond to questions from the previous lesson, leaving only a few minutes of the forty minute period for new lesson development. As a result Carl only has time to convey rules and procedures so that students are in a position to do their homework assignment. Carl does not think that a forty minute period allows him to incorporate all the elements of a good lesson. Consequently, his teaching approach is mechanical in nature, beginning with answering several questions from the previous lesson, followed by a brief lesson on a narrow skill or concept, then demonstrating one problem and finally leaving less than ten minutes for students to practice. On several occasions, students did not ~~have~~ any practice time in class because the time was used to answer questions from previous lessons. Carl's teaching and preparation for teaching is not only guided by the amount and type of material to be covered, but also his strongly held value for pedagogical efficiency. For Carl, pedagogical efficiency has its exclusive focus on the time factor. His concern with time is present during the planning for each lesson, when selecting a teaching strategy, when evaluating students, when

setting student expectations and when deciding on the depth and breadth of his lessons. For example, he is not prepared to use teaching strategies that lack efficiency from a time point of view and therefore his classroom practice is dominated by drill and practice, teacher demonstrations and expository teaching. The use of a single text, the use of the multiple choice format on his quizzes and tests and restricting his program to the core curriculum are further examples of the time constraint experienced by Carl. However, the observations seem to suggest that the students, for the most part, are quite comfortable with the drill and practice routine - working productively and showing a willingness to learn and acquire the mechanical steps in solving problems. Furthermore, students seem quite comfortable asking Carl questions, but they were not always comfortable with the responses received. On one occasion a student asked: Is $\sin 30^\circ = 0.5000$ degrees or radians? The response was a simple "No" without explaining why neither was correct. Neither the teacher nor the student pursued this any further, but it was obvious that this student and others did not understand the response. Following the class period, Carl realized that he should have given a better explanation, but he never thought of it at the time. These kinds of incidences occurred on several occasions during his lessons and they appeared to be symptomatic of the time pressure felt by Carl.

The students in Carl's classroom experienced a number of frustrations stemming from their own lack of skills. This was particularly obvious in the area of algebraic manipulations (eg. $1/\cos \theta + 1/\sin \theta$). As Carl illustrates problems to his class, he is always raising questions for students to answer. However, the questions seem to be limited to what Carl is comfortable with or what is seen to be essential for solving the problem. His problem-solving activities seem to be driven by three underlying objectives: to arrive at the correct solution, to clarify the mechanical solution process and to make students aware that this problem could be on the MDE. Problem-solving indicators, like understanding the problem, devising a plan, carrying out the plan and looking back, are rarely used by Carl and only then when he deals with trigonometric applications. For

the most part, the student activities in Carl's classroom consist of passively following illustrations by him on the blackboard, copying notes and solved problems from the blackboard, seeking clarification/explanations from him, and doing practice exercises. Students who experience difficulties receive individual assistance from Carl during classtime or he groups students so they could assist each other. Despite the nature of his classroom practices, there is considerable interaction between Carl and his students during the teaching process. However, it is not uncommon for him to resort to the lecture approach, particularly when the concept is perceived as difficult by him. The following comment reveals Carl's thinking.

"Sometimes when I see a difficult concept coming up, something that I have to get across and then I will use a different approach. I will go to the lecture type ... saying, this is the way it is ... it really ... there isn't really an easier way to do this."

Having students accept mathematical information or knowledge on face value seems to be Carl's way of dealing with the pressure stemming from the amount of content to be covered. This is again evident with respect to his treatment of trigonometric formulas. More than one half of the twenty seven formulas were not covered at all, ten he stated and used without any justification and for two he gave a formal proof. Carl feels that as long as students can apply them he is satisfied. He sees no need in memorizing information unnecessarily, particularly when this information is provided to students in a data booklet during the MDE.

There is considerable interaction between students in Carl's class. He is not opposed to students working together because he recognizes that "...quite often they can pick it up much better from each other than they can from me." He also acknowledges that the students, who are doing the explaining, often solidify their own grasp of a particular concept. Carl fosters a cooperative attitude in his classroom in that he will not accept students putting each other down.

Carl admitted that he is not a believer in the problem-solving approach to teaching. Although he professes to personally liking it and using it below the grade twelve level, he finds it very difficult to teach. A poster, displaying the four problem-solving steps, is located beside the blackboard which is rarely referred to during his teaching. Carl explains that the lack of emphasis on problem-solving at the grade twelve level is due to a lack of time, but he also feels that these students know "how to go about it." The classroom observations and the other sources of evidence suggest that Carl believes that students must master the skills and concepts first before they engage in problem solving. As a result, Carl treats problem solving as a separate activity at the end of the unit where students have an opportunity to apply their knowledge to real life application problems. The practical applications selected by Carl are almost exclusively from the student text. He sometimes modifies a problem so that it reflects a situation that students can identify with. However, he feels that students do not see these applications as important or as having any practical use for them. This perception has caused Carl to shift in focus. He says:

"...I approach it more with the idea that what they are doing here in the mathematics, especially when they have a problem-things of that nature, is to develop their mind, making it possible for them to think through a type of problem, to be able to go from...problem through to the end...through to be able to arrive at what the question was."

Rather than worry about the students' perception of problem solving, Carl simply views this activity as an opportunity to practice mathematics and develop the students' minds. He is strongly concerned about the students' understanding of mathematics and their success on the MDE. However, the evidence seems to suggest that his concern for the students' understanding of mathematics is strictly related to the skills and concepts necessary to pass the MDE and not mathematics in general. This focus on student understanding as it relates to the MDE is also evident from the nature of the quizzes and tests. Multiple choice tests are administered because the bulk of the MDE is of the

multiple choice format. Carl seems to be preoccupied with the MDE in that he makes constant reference to it in his class. Carl's objective is to teach the core curriculum well, "...so that students understand it and are ready for the exam." The impact of the MDE on Carl's classroom practices is readily admitted by him, when he says:

"...truthfully, I am trying to teach them how to do the departmental exam...writing of that thing...that's what is more important to them than anything else that there is...is to be able to get through, get a good mark and that's it."

Carl shows much empathy with the students' anxiety about the MDE which is reflected in his classroom practices. He shows it by reviewing the content which includes the writing of previously written MDE under test conditions. The tests are then marked, difficult problems are reviewed and demonstrated and an attempt is made to increase the comfort level of students about writing the official MDE.

Carl finds that keeping students on task and interested in mathematics is sometimes a real challenge. On occasion he is concerned about their productivity, a problem that he feels is caused by the program being offered on a ten-month basis. A shorter time period, like semestering, would make the course more intense which would require students to apply themselves. Closely connected with this concern is his teaching load. Currently he teaches seven different courses which is a critical factor affecting his teaching. He is deeply concerned about his lack of time for Math 30 because the other courses require his attention as well.

There were repeated incidents that suggest that Carl places considerable value on being professionally competent. He consciously avoids dealing with content when he is unsure or had an unsuccessful experience with it in the past. His treatment of the concepts of amplitude, periodicity and phase shift is a case in point. He explains his superficial treatment of these concepts as follows:

"...I don't really feel comfortable to do that because I don't know enough about physics or economics or music for that matter."

This avoidance is again displayed with respect to computers. Carl is not prepared to use them because he has no computer background nor is he familiar with any software. He feels more comfortable with calculators and consequently they are used in his classroom. In identifying the factors that influence his selection of curriculum materials for use in his classroom, Carl remarked that if he is uncomfortable with the content or if he had an unpleasant experience with it in the past, he would not use these materials in his lessons if he could help it.

Carl has clearly stated expectations of his students. He expected students to work hard and apply themselves in class. While he gave only minimal homework in terms of specifically assigned exercises, he expected students to do homework every night in areas that they felt they were weak in. In terms of content as well he has clear statements of expectations. With respect to trigonometry, he expects that students understand and remember "...the three primary trig ratios, the reciprocal trig ratios and the coordinates of the points on the unit circle." In terms of trigonometric identities, he expects "...students to apply the formulas when given", because students receive a data booklet when writing the diploma exam which contains all the trigonometric formulas needed to solve the problems. The law of sine and cosine is important to Carl and therefore he expects students to recall or recognize both formulas and then apply them.

Carl's Students

Carl values his personal relationship with each student and the personal attention that he is able to give in his class of ten students. He enjoys his students very much and describes them affectionately as a "good bunch of kids." The students like Carl as well which is apparent from the good natured bantering that goes on between himself and his students. Although his class is small, there is still a wide range of interests and abilities present. His relationship with the students is focused on their interests and abilities, both of which seem to influence his interpretation of curriculum materials for use in his classroom. He fosters the students' interest by providing applications from the students' immediate environment. Keeping up the students' interest and maintaining a productive environment in which students apply themselves and remain task-oriented requires his continual attention. Carl states:

"The most critical thing that I find in the teaching of Math 30 is to be able to cause the students to work at...continually keeping up their interest and their...their...it's not enthusiasm, but their...to make them work at the subject rather than slough it off and say: 'Oh well, I can pick that up in the last week of school or something of that nature' ...that's the hardest portion of it is to keep them with their noses to the grindstone, to keep them at the thing. Perhaps that's why a semestered type of thing is better because it is over a short period of time and you can...it is more intense, you can keep them at it, because it is for a shorter period of time. So when it is stretched over a long period of time it is more difficult to keep the students applying themselves."

He consciously limits or expands coverage of content to address these interests and abilities. Carl considers himself fortunate that in his small class he is able to assist students individually. This personalized approach to assisting students is based on his personal knowledge of each student because most of them have been at School C since kindergarten. Carl seems quite accepting of students who experience difficulties in mathematics as long as they are willing to try. He feels that all his students are capable of grasping the mathematical

concepts taught, only that some are less motivated to do so.

Students who are new to the school or are not as familiar to Carl receive his attention through observations and one-to-one contact. This allows him to determine to what extent the student grasps the concepts and understands mathematics. Carl relies much on verbal clues, oral responses, personal interaction and general reactions from students as he tries to get to know them. Students who are reticent to take an active part in his class or appear shy or withdrawn, are approached by Carl privately and with a great deal of sensitivity. When it is determined that the student experiences academic difficulties, Carl not only offers assistance in class, but he is also prepared to modify the program. This is done primarily through limiting coverage of materials, limiting the treatment of problems or limiting the use of resources to one text. In actual practice this means confinement of the program to the core curriculum, confining problem solving to one solution process and the use of one text to avoid differential treatment of topics and exposure to different terminology. While all of these measures are also used to assist Carl in addressing his time concern, he is clearly using these same measures with the conviction that they address the students' needs when they experience difficulties. This implies that the students' interests and abilities are addressed with something other than the use of multiple resources.

The strong influence of the MDE and the students' future careers on his teaching emerge again as factors in his relationship with students. Carl is quite up front with his comment:

"...of foremost importance is that the kids pass the diploma exam. Preparing them for further study at universities and technical institutions is of importance to me as well."

Carl's perception of his students is that they have a good attitude toward learning mathematics and a good understanding of the content. These perceptions are not confirmed by the results from the ATMT and MDE. With respect to Mathematics in School, the students' responses show that their

attitude is a reflection of Carl's classroom practices. Students are saying that, checking an answer to a problem by going over it, using algebraic techniques for dealing with trigonometric problems and memorizing trigonometric formulas and theorems, are very important. Carl does spend the major portion of the class period going over problems that students do not understand, consequently they view it as an important activity in the learning process. The importance that students place on using algebraic techniques for dealing with trigonometric problems seems to be an indication that students recognize their own lack of skills related to algebraic manipulations. Classroom observations confirm that several students have difficulties with the algebraic treatment of trigonometric expressions (eg. $1/\cos \theta + 1/\sin \theta$). Carl's classroom practice and his rule and skill-oriented view of mathematics seem to be reflected in the importance that students attach to memorizing trigonometric formulas and theorems. In general, the relationship between the importance that students attach to Mathematics in School and the degree to which they like it is minimal. Activities that are rated high in their importance received a low enjoyment rating and vice versa. However, the overall conclusion is that students rated the importance of Mathematics in School higher than the degree to which they like it. With respect to Mathematics as a Process, the responses suggest that not only is there always a rule to follow in solving trigonometric problems, but that these problems cannot be solved without using a rule. The students also feel that trigonometry helps one to think according to strict rules, but it also helps one to think logically. While the students agree that trigonometry is a good unit of study for creative people, they still see it as a set of rules. The dimension, Mathematics and the Student, which measured the students' personal feelings toward mathematics suggest that their attitude is relatively positive. In particular, students are saying that: "I feel good when I solve a mathematical problem by myself" and "I really want to do well in trigonometry".

Carl's assumption that his students have a good understanding of the trigonometry unit is not confirmed at all. His students achieved significantly

below (36%) the provincial average (61%) for trigonometry alone. The overall achievement for Math 30 (51%) is also significantly below the provincial average (65%). Judging by the relatively high standard deviation (21.6) for trigonometry, there is a considerable spread among the scores which can be interpreted as a measure of inconsistency of the students' performance. Aside from the wide range of abilities among his students, these achievement results seem to be closely related to Carl's classroom practices and his limited coverage of the curriculum.

The evidence suggests that Carl is influenced in his interpretation of curriculum materials by a number of student related and other factors. Among these are: what students will need to know in order to succeed in their post-secondary education, what students will need to know in order to do well on the MDE, what students require in order to understand the structure of mathematics, what students will enjoy and find personally interesting, the students' ability, the guidelines given in the Math 30 curriculum, the time available, approaches that have worked well for me in the past and the student text.

Summary Statement of Carl's Case

Carl has a strong mathematics background and extensive experience teaching mathematics. However, his students are not receiving the full benefit from his background and experience. This is partly due to the fact that he is reticent to display his mathematical strength fully and thus open opportunities for himself and his students to explore. Over the years he developed a view of mathematics that places an emphasis on remembering skills, rules and procedures to be applied when solving problems instead of conceptual understanding of mathematics. This view of mathematics is also reflected in his classroom practice which views teaching as being equivalent to "telling" students. It is not entirely clear how Carl came to adopt this view, but what is clear is that the MDE has a powerful impact on his classroom practices and his

interpretation of curriculum materials for use in his classroom. His constant preoccupation with the MDE seems to indicate that this exam has become the end toward which he teaches. Carl is certainly committed to having his students succeed on the MDE and in their future careers and he is possessed with single determination to achieve that. The teaching strategies that Carl uses are predominantly product oriented, consisting primarily of drill and practice activities at the skill level. Whether or not the MDE is responsible for this product orientation is not clear, but according to Carl it certainly reinforces this orientation. The MDE and the demanding teaching assignment seem to constrain Carl in a number of ways. His acute lack of reflection time does not allow him to bridge the intended curriculum with appropriate classroom practices. The curriculum objectives of the intended curriculum are interpreted by Carl at the skill level, hoping to provide students with a "bag of tricks" that will help them succeed on the MDE. Except this "bag of tricks" does not contain all the tricks because Carl feels that teaching students one thing well is better than teaching them too many things poorly. This view places students at a real disadvantage in terms of their ability to deal with a variety of problem situations.

The MDE, in a sense, legitimates Carl's approach and it allows him to attend to his lesson objectives as the most important priority. In fact, the observations show that Carl has a single determination about achieving his objectives and he is not always appreciative or accepting of the students' need to know. Evading student questions or responding with answers to questions the students have asked are typical behaviors displayed by Carl.

In general, Carl seems to feel much more comfortable in the role as disseminator of information than as a facilitator of learning. Instead of organizing his class that allows students to explore mathematics, to find out for themselves and to make sense of the concepts within the overall mathematical structure, Carl would provide students with information, tell them right from wrong, demonstrate examples and ensure that students engage in sufficient drill and practice activities. Inherent in the view of the teacher as a disseminator of knowledge is the underlying issue of trust and control. Trust in a sense that

students are capable of producing knowledge, capable of learning on their own and capable of understanding, given the right support structures in the classroom. The issue of control, in Carl's case is related to him being in charge and knowing best what students need and when they need it.

Carl seems aware of most of the influencing factors relative to his view of school setting, school mathematics, teaching and students.

C. Summary of CPTT Survey Responses (6 Teachers)

In order to situate the three cases studied with other teachers teaching the same unit, six Math 30 teachers were selected from six different geographic regions across the province and they were asked to respond to a "Classroom Practices of Trigonometry Teachers" survey (CPTT). The following question was posed and answered using the results of this survey: What are the provincial trends with regard to the four curriculum commonplaces: school setting, school mathematics, teaching and students?

School Setting

The teacher sample consisted of five male and one female teacher, ranging in age between 36-45 years for the male teachers and 26-35 years for the female teacher. All teachers are mathematics specialists, except for one who specialized in general science. In terms of teaching experience, the responses indicate that all teachers have at least ten years experience in the classroom. The majority of the teachers has more than fourteen years teaching experience. Their experience in teaching high school mathematics is also more than fourteen years for the majority of teachers, only one teacher reported six to nine years. Each one of the teachers taught at least four different courses at the time and one teacher taught as many as seven different courses. These courses were not all in the area of mathematics as was indicated on the survey. Two teachers taught 60 to 79% mathematics and four teachers taught it 80 to

100% of the time. The number of different students who were taught by the teachers ranged from 50 to 150 students. However, the majority of the teachers taught fewer than one hundred students. Although nearly all teachers were mathematics specialists, their mathematical background varied considerably. The number of full-time mathematics courses (including Statistics and Computing Science) ranged from 3.5 to more than 7.5 courses, with an even distribution of teachers over that range. The range for the number of Curriculum and Instruction courses in mathematics taken by teachers ranged from one to three full-year courses. The adequacy of the teachers' background as it relates to the teaching of elective units, problem solving, mathematical applications, statistics and the implementation of the overall philosophy, goals and objectives of the Math 30 Curriculum, can be described as generally ranging from good to very good. However, even based on this small sample, it is evident that there are teachers who consider themselves barely satisfactory and even unsatisfactory in some areas. Teachers rated the physical setting in their school in which Math 30 was taught as good to satisfactory. Although the six schools were located in communities ranging in size from less than 1000 to 30,000 people, none of the schools enrolled more than 1000 students. However, the grades offered in each school included the following combinations: K-12, 7-12, 9-12, and 10-12. There was also a wide range of staffing levels and the number of mathematics teachers evident across the schools. These school setting factors have often a bearing on the program and its outcome in that they may determine the level of resources, the nature of the teacher's assignment, the degree of professional development or the teaching strategy used. Examples of this would be the absence of library resources, the unavailability of computers or the nonexistence of a math-lab. Given any or all of these conditions, teachers would very likely not employ teaching strategies that involved these facilities or resources.

The teachers' responses to the Skill Emphasis Survey (SES) also indicate that the use of electronic devices (eg. calculators and computers) for solving problems in trigonometry is of primary importance in their classroom.

Such use, at least by teachers, on an occasional basis was again confirmed by their responses to the Teaching Strategy Survey (TSS) and the Resource Material Survey (RMS).

School Mathematics

An examination of the responses to the Trigonometric Subtopic Questionnaire (TSQ) revealed that subtopics were given one of five different treatments: (1) taught for the first time, (2) reviewed and then extended, (3) reviewed only, (4) assumed as prerequisite knowledge and (5) not taught and not assumed as prerequisite knowledge. Out of the thirty two subtopics there was only complete agreement about the treatment on twelve. For all other subtopics the treatment varied over the full range among the six teachers. This was particularly the case when subtopics were not specifically stated objectives of the curriculum. Where the subtopic was also a required curriculum objective, the range of treatment covered the first three with one exception. This means that students dealt with the objectives at least to some degree. The exception to this is the subtopic which asks teachers to derive and apply the complimentary arc formulas: $\cos(\pi/2 - \theta) = \sin \theta$ and $\sin(\pi/2 - \theta) = \cos \theta$. Five teachers indicated they taught it for the first time and one teacher assumed it as prerequisite knowledge. In stating the first and second primary reasons for teaching or reviewing any of the subtopics, the overwhelming number of responses converged on three reasons: (1) it is included in the Mathematics 10-20-30 Curriculum Guide (1983), (2) it is useful for further study in mathematics and (3) it is assumed to be on the MDE. Three of the subtopics were not taught and not assumed as prerequisite knowledge by two teachers. These teachers stated that the subtopic was not in the Mathematics 10-20-30 Curriculum Guide (1983), was not useful for further study in mathematics, was not emphasized in the student text, was not assumed to be on the MDE and it was disliked by the students.

The responses to the Trigonometry Objectives Emphasis Survey (TOES)

suggest that maintaining previously developed skills, finding the domain and range of the six trigonometric ratios, solving problems involving areas of regular polygons, and describing circular paths using the initial point and directed distance of the path receive the least amount of emphasis. The objectives receiving the most emphasis include: define radian measure, solve oblique triangles by using the sine law and/or cosine law, apply the sine law and cosine law to practical problems, solve simple trigonometric equations and give the value of one of the trigonometric ratios, evaluate the other five trigonometric ratios. On a five point scale the range of the mean ratings is from 3.17 (the lowest) to 4.83 (the highest) and it suggests that all objectives are receiving medium to heavy emphasis. However, the sample data also reveal that there is a considerable range of emphasis on several objectives as is indicated by the larger standard deviation.

The Trigonometric Formula Questionnaire (TFQ) revealed that teachers presented formulas in a variety of ways. For only two of the twenty seven formulas, all teachers presented them in the same way to their class. In some instances the teacher responses covered the full range of options for presenting a formula to students. In some classes the teachers gave a formal proof for a formula and in others the teachers did not cover the formula or discuss it with the class. However, the majority of responses seem to suggest that most of the formulas covered were either formally proven or stated with an informal justification.

The responses indicated that the primary reason for providing a formal proof was that the proof unifies basic ideas. The second major reason cited was that the proof is an interesting example of mathematical reasoning. With regard to the teachers' expectations of student mastery for the various formulas, again, the responses cover the full range of options. The range being: (1) prove and apply formulas, (2) informal justification and apply, (3) recall and apply formula, and (4) apply formula when given. The responses indicate that for the majority of instances (42%) teachers expect that students are capable of providing an informal justification of the formula and then apply it. Recalling

and applying the formula ranked second (30%) and the remaining two options were nearly equal.

Of the four trigonometric interpretations, teachers actually used only two - the ratio interpretation and the unit reference circle interpretation. The wrapping function interpretation and the line representation were used by 67% and 33% of the teachers, respectively. With respect to the ratio interpretation, teachers stated a wide variety of reasons for using it. The reasons ranged from it being emphasized in the Alberta Curriculum Guide, being enjoyed and easily understood by students to being on the MDE. Reasons for using the unit reference circle interpretation included: easy for students to understand, useful in the study of mathematics in the future and the interpretation is on the MDE. Those teachers who included the wrapping function interpretation cited the same reasons as those cited for the ratio interpretation. One teacher indicated that this interpretation was not used because of lack of knowledge about it. For the line representation, the reasons for including it were again similar to those stated for the other interpretations. Teachers who did not use it indicated that they did not know this interpretation well, it was not emphasized in the Alberta Curriculum Guide and it is difficult for students to understand.

The responses to the Skill Emphasis Survey (SES) identified two skills of primary importance in the teachers' classroom: Skill in solving for missing parts of right triangles and skill in applying trigonometry to application problems. This is closely followed by "skill in algebraic techniques for dealing with trigonometry problems", "skill in solving equations involving trigonometric identities", and "skill in the use of electronic devices (eg. hand calculators, micro computers) for solving problems in trigonometry". Of least importance to the teachers was the skill in graphing inverse trigonometric functions. The skill in solving for missing parts of right triangles was of primary importance to four teachers and two indicated that students already had this skill mastered.

The Curriculum Material Selection Survey (CMSS) revealed that there are a number of factors that influence the selection of curriculum materials for use in the classroom and consequently the teacher's view of mathematics.

Teachers identified "what students will need to know in order to do well on the mathematics diploma exam" as an extremely important influence. Other factors such as: the guidelines given in the Math 30 Curriculum Guide, the students' ability, the time available, approaches that have worked well for me in the past and the material builds on or extends concepts covered in previous grades (or activities) followed closely. Still important, but of lesser importance than the six factors cited above are: What students will need to know in order to succeed in their post-secondary education, what students require in order to understand the structure of mathematics, what students will need to know in order to understand the importance of mathematics in their daily life, what society needs by way of informed citizens, the material is emphasized in the student text and my own interests, background and preference. The remaining factors which include: What students will enjoy and find personally interesting, what students find easy to understand and what students require in order to understand the historical development of mathematics approached neutrality in terms of their mean ratings and they were viewed as having minimal or no influence on the selection of curriculum materials.

According to the Time Allocation Questionnaire (TAQ), teachers spent an average of 29 hours (S.D. 4.4) on the study of trigonometry in the Math 30 classroom. The suggested time allocation in the Curriculum Guide is 26 hours for the study of trigonometry. The total number of hours used by the teachers ranges from 25 to 35 hours which is an acceptable deviation from the suggested time allocation. However, the responses to the twelve major topics or activities that comprise the trigonometry unit reveal that there is a wide range of time spent on each. To illustrate, the time spent on the unit circles ranges from two to eight hours, the time spent on the study of trigonometric identities ranges from one to seven hours and the time spent on developing elective topics ranges from zero to five hours. While it is recognized that the topics or activities are not mutually exclusive, the responses do suggest, however, that teachers do interpret the curriculum in a wide variety of ways as is evident from the instructional emphasis given to the various topics.

Teaching

The responses to the TSQ suggest that teachers value student understanding and success on the MDE. The inclusion of topics in their lessons is guided very much by the Mathematics 10-20-30 Curriculum Guide (1983), by the MDE and by their usefulness for further study in mathematics.

Teachers seem to value high student expectations and academic rigor as is reflected in the responses to the TFQ. The majority of formulas are formally proven or stated with an informal justification. However, for the most part, students are expected to provide an informal justification for the formula and apply it or simply recall the formula and apply it. In fourteen percent of the instances students are expected to prove formulas and then apply them. This requirement goes well beyond the expectations set out in the Curriculum Guide.

The responses to the TIQ seem to suggest that the use of a particular interpretation depends on whether it is in the Mathematics 10-20-30 Curriculum Guide, is on the MDE, is useful for further study of mathematics, is enjoyed by students and easily understood by students. Two trigonometric interpretations were excluded by some teachers because they did not know these interpretations well. This seems to suggest that the teacher's own background and professional competence are significant factors in the selection of curriculum materials for use in the classroom.

The Teaching Strategies Survey (TSS) suggests that teaching is characterized by pedagogical efficiency. The responses indicate that lectures, teacher demonstrations, class discussions (ie: teacher/class dialogue) and drill and practice are the prevailing teaching strategies used by teachers. In addition, students receive worksheets, they write tests and quizzes and they do varying degrees of homework. The use of computers by students occurs never or rarely, however, teachers use the computer on one or two occasions in this unit. Teaching strategies, such as individual projects, library research, math-lab experiments, field trips, resource persons, film or videos, newspaper articles, class projects and individual learning packages occur never or rarely in

classrooms. However, the responses suggest that there is a small portion of teachers who are beginning to focus consciously on problem solving of "real life" applications, who look for applications of trigonometry to other curricula or subjects and who incorporate topics from other subjects into the trigonometry unit.

The selection of curriculum materials is closely linked with effective teaching and it is influenced by a number of factors which is evident from the responses to the CMSS. The strongest influence on the selection of curriculum materials and ultimately the teaching strategies comes from the MDE. A close second factor is the Curriculum Guide, followed by the students' ability, the time available, approaches that have worked for the teacher in the past, the requirement that the material builds on or extends concepts covered in previous grades (or activities), what students will need to know in order to succeed in their post-secondary education and what students require in order to understand the structure of mathematics. The remaining seven factors had a lesser influence on the selection of curriculum materials for use in the classroom.

The responses to the Resource Material Survey (RMS) indicate that the text Math IS/6, the Curriculum Guide and teacher-prepared worksheets, exercises and problems are the most frequently used resources by teachers. This seems to be indicative that teachers adhere closely to the prescribed core curriculum in order to prepare students well for the MDE. However, the responses also suggest that teachers are drawing, at least occasionally, on other textual resources to supplement their program.

The responses to the Time Allocation Questionnaire (TAQ) provide another insight into how the trigonometry unit is taught. While the average total time spent on the unit is 29 hours, the range is 25-35 hours. If the time spent on selected topics or activities that make up the unit is an indication of the instructional emphasis given to these topics by teachers, then the responses indicate that there are vast differences among teachers relative to the perceived importance of each topic or activity and the interpretation of the curriculum.

Students

The teachers describe the general attitude of their students toward learning Math 30 as good to satisfactory. They also feel that their students have generally a good to satisfactory understanding of the material in the trigonometry unit. One teacher described the level of understanding as very good. The students, their understanding and enjoyment of and ability in mathematics, seem to be a relatively strong focus as is indicated by the responses to the TSQ, TIQ and CMSS.

V. INTERPRETING THE MATHEMATICS CURRICULUM

In this chapter the findings related to the first three major research questions are reported using the case reports.

A. Teachers' Interpretations

The interpretation that teachers give to the intended curriculum is the focus of the following question.

Question 1: **What interpretation do teachers give of the intended curriculum as represented by the trigonometry unit of the Mathematics 30 curriculum?**

This question was examined by focusing on the classroom level where the intended curriculum, as reflected in the Mathematics 10-20-30 Curriculum Guide (1983), was taught by teachers. The question became one of contrasting the teacher interpreted curriculum against the intended curriculum.

From the three cases that were studied using observations, interviews, teacher and student survey and the results of the MDE, it is evident that teachers share common perceptions about the intended curriculum. However, these perceptions seldom result in common practice, suggesting that they are quite different or in different locations on the perception continuum. TABLE 1 contains a summary of the elements of the intended curriculum and the corresponding findings of the teacher-interpreted curriculum (ie. implemented curriculum). The contents of TABLE 1 are based on an analysis of the intended curriculum and the data for the three case studies.

TABLE 1

**Summary of Elements of Intended Curriculum for High School Mathematics
and the Teacher Interpreted Curriculum for Trigonometry**

INTENDED MATH CURRICULUM**TEACHER-INTERPRETED
CURRICULUM****1. Goals of Senior High Math Program:**

They are stated in relation to three main expectations and needs:

Student Development

- * To develop in each student positive attitudes towards math.
- * To develop an appreciation of the contribution of math to the progress of civilization.
- * To develop the ability to utilize math concepts, skills and processes.
- * To develop the powers of logical analysis and inquiry.
- * To develop an ability to communicate math ideas clearly and correctly to others.

- * Degree of achievement varies with teacher. Not achieved with all students.
- * Limited attention given, only incidental. No reference to the historical development of trigonometry is made.
- * Primary focus of all three math teachers. Driven by MDE.
- * Considered important, but emphasis varies with teachers.
- * Limited attention given, varies with teacher. Much emphasis by teachers on producing the product and not talking about the process.

Discipline of Mathematics

- * To provide an understanding that math is a language using carefully defined terms and concise symbolic representations.
- * To provide an understanding of the concepts, skills and processes of math.
- * To provide an understanding of the common unifying structure in math.
- * To furnish a mode of reasoning and problem solving and using these in practical situations.

- * Teachers pay much attention to the language, definitions and symbols being consistent with those used on the MDE.
- * Primary focus of all three math teachers, but their success varies.
- * Emphasized by some and not by others. Significant variance among teachers.
- * Teachers at different levels on the continuum. Gaining in importance among all teachers.

Societal Needs

- * To develop math competence in students to function as citizens in today's society.
- * To develop an appreciation of the importance and relevance of math as part of the cultural heritage that assist people to utilize relationships that influence their environment.
- * To develop an appreciation of the role of math in man's total environment.

2. Program Rationale:

- * Math as an important component of public education and essential to function effectively in society.
- * Math as formal study of mathematical concepts and relationships must be in balance with applied use in every day situations.
- * Math as part of basic education of students at all levels must ensure continuity and yet be responsible to emerging influences in our changing times.
- * Math programs should have flexibility in selection of topics to meet needs of the students.
- * Math programs should integrate the use of calculators and computers in the instructional process.
- * Math programs should emphasize problem solving, integration of microelectronic technology and applications.

3. Program Structure

- * Math 30 has core elective component.
- * Core component represents minimum educational objectives prescribed for all students in program.

- * Some emphasis given by all teachers. Results vary within classes as reflected on the MDE.
- * The ATMT results suggest that teachers are developing an appreciation of the importance and relevance of math in most students.
- * Limited emphasis. Teachers at different levels of the continuum.

- * Viewed as important in its own right but also in relation to other disciplines.
- * Emphasis is on practicing skills, procedures and routine tasks and limited emphasis on applied use to everyday situations.
- * Trigonometry viewed as an important part of the math curriculum and an appropriate continuation of previously taught material.
- * Rigid adherence to core content because of MDE. Limited extension of content to meet needs.
- * Calculators used by all students, primarily for checking work or solving problems. Limited use of computers.
- * Problem solving receives increased attention with limited use of microelectronic technology and applications.

- * Offer core component only.
- * Closely adhered to. Sometimes exceeded with personal list of topics. Some content ignored because it is not assumed to be on the MDE.

- * Elective component allows variety and flexibility in choice of topics and structure.

- * Core and elective are both mandatory requirements.

- * Math 30 emphasizes theoretical development of mathematical concepts and relationships through deductive reasoning.

4. Program Emphasis

- * Problem solving skills development.

- * Applications, mathematical concepts and skills applied to real life situations.

- * Integrate problem solving and applications into overall program instead of independent treatment.

- * Calculators and computers encouraged.

- * Trigonometry has 19 specific objectives.

5. Program Planning

- * Topic and objective sequence are flexible.

- * Teacher planning encouraged.

- * Elective component for Math 30 to be 20 hours to provide opportunity to spend time on interesting and useful areas of math.

- * Elective component very limited. Time is used on core topics.

- * Limited to core curriculum. Elective component not part of the MDE. Therefore very limited treatment in class.

- * Teachers at different levels of the continuum. There is some attention given to conceptual understanding, multiple connections and problem solving which is followed by a general tendency to emphasize memorization of facts and procedures and to increase proficiency with paper-and-pencil skills.

- * Observed in classroom, receiving some attention by all teachers, but the teachers are at different levels on the continuum. Still treated as a separate activity.

- * Applications confined to those shown in the student text in most cases. Some applied use to other subjects and disciplines evident.

- * Still treated as an activity separate and apart from the overall program. Varies with teacher.

- * All students are using calculators. Limited use of computers.

- * Interpretation varies with teacher and it is tailored to what is assumed to be on the MDE.

- * Teachers follow chosen sequence, but decide to deviate when necessary.

- * Teacher planning occurring. (Long range, unit and daily plans are kept), with focus on core curriculum.

- * Time used to teach core curriculum and on occasion used to extend core topics.

- * Elective component is to take the interests and abilities of students into account.

- * Elective component should be part of the evaluation of students

- * Elective component designed to be a motivational aspect of math program.

- * Elective component should be utilized to challenge the academically talented students.

- * Suggested time allocation for trigonometry unit is 26 hours.

- * Elective component very limited, the time is used to teach almost exclusively core content.

- * Not evaluated at school level, because it receives very limited attention by teacher and no attention on MDE.

- * Only extended core content used to motivate students.

- * Only extended core content used to challenge academically talented students.

- * Wide range of time allocations observed for trigonometry (25-40 hours).

6. Learning Resources

- * Prescribed reference: Math Is/6 and Foundations of Mathematics for Tomorrow: Senior.

- * Recommended reference: Holt Mathematics 6.

- * Math Is/6 used as the primary student text. FMT used as additional reference. Text defined boundaries for math taught.

- * Used as secondary reference. Some use was made of resources beyond the textbook for either content or teaching method.

Goals of Senior High Mathematics Program: TABLE 1 suggests that not every goal is consciously adhered to in the trigonometry unit. The goals that deal with the development of the students' ability to utilize mathematical concepts, skills and processes and providing students with an understanding of these goals are receiving considerable emphasis. The findings further suggest that the goal dealing with modes of reasoning and problem solving in practical situations is gaining in importance in that teachers are making a conscious effort to incorporate activities and instructional techniques that are supportive of this goal. While teachers are at different levels of the continuum, they are valuing students' understanding of the common unifying structure of mathematics by placing an increased emphasis on the importance of using connections among mathematical topics and between mathematics and other

disciplines.

There appears to be a direct relationship between the teachers' emphasis on specific goals and the degree to which they are measured by the MDE. The goals that are not directly measurable by the MDE or that aim at outcomes projected to the future received less attention than those measured by the MDE.

Program Rationale: The program rationale of the intended curriculum contains six major statements that provide the philosophical framework of the high school mathematics program. These statements are also consistent with the beliefs of a number of writers (Robitaille & Dirks, 1982; Usiskin, 1980; NCTM: An Agenda for Action, 1980; NCTM: Curriculum and Evaluation Standards for School Mathematics, 1989). Based on the classroom observations, interviews and surveys, it is evident that the teacher-interpreted curriculum for the trigonometry unit is not entirely consistent with the program rationale in all areas. Incongruence with the rationale is particularly obvious in three areas. The intent is that the mathematics program be flexible in the selection of topics to meet the needs of the students, yet, for the most part all three teachers adhered rigidly to the core content with very limited provision of elective topics. Only limited extensions of core topics occurred for the purpose of challenging some students and making it interesting. The integration and use of calculators and computers in the instructional process is another area that is incongruent with the intended curriculum. While calculators were used by all students in all three classrooms, their use, however, was primarily for the purpose of checking work or performing calculations. Computer and calculator-based explorations and investigations of problems and applications were not observed in the classrooms except for Alice's limited use of computers to teach graphing of trigonometric functions.

Teaching mathematics as formal study of mathematical concepts and relationships in balance with applied use in everyday situations is another rationale that was only partially met by the teachers. While there are differences among the teachers, the findings of the three cases suggest that

there is generally an emphasis on practicing skills, procedures and routine tasks and only limited emphasis on the applied use of mathematics to everyday situations. The teacher responses suggest that this interpretation is driven by the MDE.

Program Structure: The intended program structure of the core curriculum emphasizes the theoretical development of mathematical concepts and relationships through deductive reasoning. The three cases varied considerably in this area. While all three teachers gave some attention to conceptual understanding, multiple connections and problem solving, they were all at different levels of the continuum that emphasized memorization of facts and procedures and becoming proficient with paper and pencil skills.

The core curriculum contains nineteen educational objectives representing the minimum content prescribed for all students in the program. With minor exceptions, close adherence to these objectives was observed by all three teachers. In some instances content was omitted because it was assumed not to be on the MDE. In other instances content was extended with personal lists of topics that were deemed important by the teacher or they were the teacher's "pet topics". The program structure follows a core elective format and both, the core and elective component, are mandatory. The elective component allows for variety and flexibility in choice of topics and structure, providing opportunities for students to spend time on interesting and useful areas of mathematics not contained in the core curriculum. In all three cases, teachers limited the trigonometry to the core curriculum. A lack of time to cover the core curriculum properly and the fact that the elective component is not part of the MDE were two reasons cited for ignoring the elective component.

Program Emphasis: The emphasis within the trigonometry unit is intended to be in three major areas: (1) the nineteen objectives of the core curriculum, (2) problem solving and (3) applications. With respect to the objectives of the core curriculum, the interpretation varies among the three cases in terms of depth, breadth and instructional approach. However, in each case there was a

tendency to tailor the content to what was assumed to be on the MDE. Development of the students' problem solving skills through experiences that are mathematically related and relevant to everyday situations was intended to assume greater significance and importance in the classroom. Furthermore, problem solving was to be integrated into the overall program thus becoming synonymous with doing mathematics. It was to become the process by which the fabric of mathematics is both constructed and reinforced. (NCTM: An agenda for action: Recommendations for school mathematics of the 1980s, 1980) Again, while problem solving received some attention by all three teachers, it ranged from treating it as an independent activity for Carl to nearly achieving the intended curriculum for Bill. The third area of emphasis, applications, is intended to make the study of mathematics both meaningful and relevant to students. Like problem solving, applications are to be integrated into the overall program rather than treating them as independent activities. The curriculum intends that teachers draw problems from a variety of contexts and disciplines as concepts are developed in the instructional process. The use of calculators and computers is strongly encouraged in that process. The interpreted curriculum as observed for the three cases differed markedly with respect to applications and use of technology. Carl confined the use of mathematical applications exclusively to the student text with an emphasis on practicing the newly taught concepts, skills and procedures. Scientific calculators were used by students to work out solutions, but computers were not used at all. Alice, on the other hand, incorporated applications beyond the student text, drawing from other resources and using applications to introduce new mathematical content. In addition to using calculators, Alice also used computers during the lessons on graphing. Although Bill's use of technology was confined to scientific calculators, the use of problems and applications to teach new mathematical content was extensive. Bill believes that this approach assists in developing a better understanding of concepts and procedures and it provides opportunities to review processes already learned.

Program Planning: Teacher planning is encouraged and is occurring, but with an almost exclusive focus on the core curriculum. The findings suggest that in all three cases, the sequence of topics and objectives to be taught is carefully arranged in accordance with what worked best in the past and the nature of the students. The suggested time allocation for the trigonometry unit is 26 hours. In addition, a total of 20 hours is available for the elective component of the Math 30 course from which some or all of the time could be used to develop elective topics in trigonometry. The intents behind the elective component are to provide opportunities to spend time on interesting and useful areas of mathematics, to provide for the interests and abilities of students, to be a motivational aspect of the mathematics program and to challenge the academically talented student. The three case studies revealed that there is a wide range of time spent in the trigonometry unit. Alice spent 25 hours, Bill spent 40 hours and Carl allocated 30 hours. The interests and abilities of students, the motivational aspect and the needs of the academically talented students are addressed in all three cases through extensions of the core content and not the inclusion of elective topics.

While the mathematics curriculum identifies the elective component as mandatory and as part of the evaluation of students, it is neither offered nor evaluated by the three case study teachers because it is not included on the MDE.

Learning Resources: The curriculum guide identifies two prescribed references: Math Is/6 and Foundations of Mathematics for Tomorrow: Senior and one recommended reference: Holt Mathematics 6. The data reveal that while all three case study teachers use Math Is /6 as their primary reference and student text, the use of additional resources differs widely.

B. Implementation Success

The focus of this question is on the processes and practices used in the classroom and it links these to the attained curriculum. The classroom practices of each of the three case teachers are examined and the results from the ATMT and the MDE are reported.

Question 2: What are the classroom practices of teachers and to what extent are these practices successful?

The answer to this question is based on the findings from the three case studies and an examination of a number of key components. The focus is on classroom processes and those elements that have either an enhancing or diminishing effect on student outcomes.

Alice: The instructional practices for Alice do not vary much from day to day and they follow a well established routine consisting of reviewing materials from previous lessons, tests, homework or problems, lesson development and student practice. Part of Alice's routine is to check homework by solving all problems on the blackboard with students participating in the process. Her students attach a great deal of importance to this routine as has been confirmed by their responses to the ATMT (Mathematics in School). This finding is also consistent with other studies (IAEEA, 1985; Robitaille, O'Shea & Dirks, 1982). Typical teaching strategies for Alice include teacher demonstrations, class discussions, drill and practice and, less frequently, lectures. This normally involves Alice interacting with her students as she leads them through an explanation of a particular concept or a solution process for a problem. Although she is under considerable time pressure, she is very conscious of the importance of connecting current content with previously learned concepts, skills and procedures. Other strategies such as small group activities, discovery approaches, math-lab experiments, library research or individual projects are rarely or never used. In general, teacher explanations, practice and directive

teaching are the most dominant strategies in Alice's classroom. However, there is evidence that problem-solving with the class as a whole does occur from time to time, but still as an independent activity. Trigonometry applications are for the most part limited to the student text and only on occasion are they drawn from other disciplines or subjects. The 25 hours that Alice allocates to the trigonometry unit is used to focus on the core curriculum and on those concepts, skills and procedures that are assumed to be on the MDE. In fact, Alice has all copies of previously written MDE and she constantly uses them for the purpose of review and student practice. Occasional extensions of core topics do occur whenever the interests and abilities of students call for them or when deemed important by Alice. Overall, the reported use of time for Alice is as follows: two tests and follow up (5 hours), trigonometric identities (4 hours), developing and applying the sine and/or cosine law (4 hours), graphing trigonometric functions (3 hours), solving equations involving trigonometric functions (3 hours), the unit circle (2 hours), reviewing and maintaining previously learned materials (1.5 hours), trigonometric functions as ratios of the length of sides of a right triangle (1 hour), finding domain and range of the six trigonometric functions (1 hour), and concept of a wrapping function (.5 hour). The concept of periodicity, amplitude and phase shift is dealt with under graphing and trigonometric applications are incorporated throughout. Alice makes no provision for developing elective topics. Given the fact that her class has 31 students and the time allocation is 25 hours (26 hours is recommended in the curriculum guide), Alice places considerable emphasis on pedagogical efficiency and maintaining a business-like atmosphere in her classroom. While there are occasions which involve student explorations (eg. using computers to explore trigonometric graphs), students spend the major portion of their class time listening to Alice's presentations, doing seatwork and taking tests. These findings are consistent with those reported in the Second Study of Mathematics (IAEEA, 1985) and by Robitaille, O'Shea & Dirks (1982). Students in Alice's class are given daily assignments of problems and exercises which they normally begin to work on in class. Students are expected to complete any unfinished work as homework for the next day. Even students who are absent

are expected to complete their assignments.

With respect to the use of instructional resources, the two prescribed references along with the Curriculum Guide stand out as the most commonly and consistently used resources and in some respects define the boundaries for the content taught by Alice. The next most common resource was the teacher prepared material (eg. worksheets, exercises and problems, tests), followed by supplementary texts. This is again consistent with other research (IAEEA, 1985; Connelly & Elbaz, 1980).

Although Alice's teaching is strongly characterized by pedagogical efficiency, academic rigor and high expectations, she maintains a friendly relationship with her students and focuses much of her efforts on their interests and abilities. She provides individual assistance in and out of class which is much appreciated by the students.

Given Alice's interpretation of the intended curriculum, the attained curriculum as measured by the ATMT seems to suggest that the students' attitude toward Mathematics in School is shaped by and a reflection of her classroom practices. For example, Alice did emphasize the recall of information, skills and procedures and consequently students held the view that memorizing trigonometric formulas and theorems is important. However, they were undecided with respect to the degree to which they liked it.

Part of Alice's daily routine includes reviewing and checking homework and assignments by going over these with the students. This activity is very popular with the students and it is also viewed as very important. The use of electronic devices for solving problems in trigonometry is liked the most by the students. This refers specifically to the use of computers and it was obvious during the observations that students thoroughly enjoyed using them. However, the importance that students attach to the use of computers is not as pronounced as the students' enjoyment level. Solving trigonometric applications is seen as important, but students were almost neutral with respect to the degree to which they liked doing this activity. The second dimension of the ATMT dealt with Mathematics as a Process. From the student responses to this questions in this dimension, it is evident that while students feel that

trigonometry helps one to think logically in accordance with strict rules, there always seem to be different ways to solve most trigonometric problems. Alice's students clearly suggest that trigonometry is a rule-based activity or process and that trigonometric problems cannot be solved without using rules.

The third dimension of the ATMT dealt with Mathematics and the Student. The responses suggest that students feel good when they solve mathematics problems by themselves. Furthermore, they indicated that they really want to do well and that they usually understand what is being talked about in trigonometry.

The attained curriculum as measured by the MDE indicates that Alice's students performed below (59%) the provincial average (64%) for trigonometry. However, when the overall achievement results of Alice's class for Math 30 (64%) are compared to the provincial average (63%), her class did slightly better.

Bill: Similar to Alice, the instructional practices for Bill follow a well established daily routine of reviewing materials from previous lessons, assignments or tests, presenting new material and providing for student practice. Quite frequently, students are asked to show the solution steps of their homework assignment on the blackboard. This allows Bill to check each problem, expose the other students to a variety of solution methods and it provides opportunities for good discussions. His students consider this to be the most important and second best liked activity in his mathematics classes. This is also consistent with other studies (IAEEA, 1985; Rosier, 1980; Robitaille, O'Shea & Dirks, 1982).

Bill makes use of a number of teaching strategies, the most frequent being teacher demonstrations, drill and practice and checking homework on the blackboard. Lectures, class discussions, student worksheets, student-directed class discussions, discovery approach, test and quizzes and trigonometry applications from other areas and subjects are often used in his teaching. Whatever strategy Bill uses, the elements of problem solving are always present. Many concepts are introduced through applications and students are encouraged to question, explore and suggest alternate solutions. Mathematics

and problem solving are synonymous for Bill. A total of 40 hours is allocated to the trigonometry unit which is nearly twice the recommended time allocation. Other than some extensions of the core topics, the time is exclusively used to teach the core curriculum with specific emphasis on the concepts, skills and procedures assumed to be on the MDE. Bill extends core topics to meet the needs of students and to cover those aspects that he considers important. The influence of the MDE is also strongly felt by Bill in that he makes constant reference to it and he uses copies of previously written MDE for review and practice. Overall, the reported use of time for Bill is as follows: developing and applying the sine and/or cosine law (6 hours), trigonometric identities (5 hours), concept of a wrapping function (4 hours), applications of trigonometric functions (4 hours), solving equations involving trigonometric functions (4 hours), reviewing and maintaining previously learned materials (3 hours), dealing with the unit circle (3 hours), finding the domain and range of the six trigonometric functions (2 hours), developing concept of periodicity (2 hours), trigonometric functions as ratios of the length of sides of a right triangle (1 hour), and developing elective topics (1 hour). The time allocated for developing elective topics is used for core topic extensions. Despite the allocation of 40 hours and the fact that his class has only 13 students enrolled, Bill still expresses concerns about time. He attributes this to his emphasis on fostering a problem solving approach with an emphasis on applications and explorations. Although Bill emphasizes a problem solving approach in his classroom, the use of computers is not part of that approach. The various instructional strategies that he uses are for the most part selected with pedagogical efficiency in mind. A good portion of the students' time is spent listening to and interacting with Bill. The remainder of the time is spent working on problems and exercises and taking tests. This is quite consistent with the observations for Alice and the findings reported in the Second Study of Mathematics (IAEEA, 1985) and in the B.C. Mathematics Assessment, Summary Report (Robitaille & Sherrill, 1977). Bill uses a number of different assessment tools aimed at measuring the students' knowledge and understanding of mathematics. In the trigonometry unit he administers two major chapter tests, numerous quizzes and he rounds out his view of where the

students are by using worksheets, class assignments, homework and extensive blackboard drills. Bill expects that students do their homework, even those who have been absent for the day. He consistently checks the homework assignments on a daily basis by asking students to write out the solution process on the blackboard while the other students check their work against it. Students rarely ever fail to do their homework.

Among the instructional resources used by Bill, the prescribed reference, Math Is/6, a text from the previous curriculum and his personally prepared materials are the most commonly and consistently used resources. Other textual resources, the curriculum guide and the mathematics materials prepared by another school system are the next most commonly used resources by Bill. His use of instructional resources follows closely the practice used by Alice.

Bill's teaching is characterized by a number of strongly held values such as pedagogical efficiency, academic rigor, high expectations, motivating students, student understanding and success on the MDE, feedback on student performance and student contributions. His willingness to provide individual assistance extends well beyond the classroom.

The attitude of Bill's students, as measured by the ATMT suggests that students view Mathematics in School as important. Checking an answer to a problem by going over it is not only viewed as the most important activity, but students also like it. This is consistent with classroom observations during which it was observed that checking problems is more than providing students with the correct answer or confirming it. Bill also views this activity as a very important component of his teaching strategy. It incorporates elements of review, teaching, remediation and extension. Solving trigonometric application problems is another activity of considerable importance to students. This seems to reflect Bill's continued emphasis on problem solving using real life applications. In general, the responses seem to indicate that the importance that students attach to the various activities is directly related to the importance that Bill attaches to these same activities. This is particularly evident from the student responses to "Using electronic devices for solving problems in trigonometry". Other than calculators, Bill does not use computers in his

classroom for any purpose and consequently students attach a relatively low rating to the degree of importance for that activity. In general, however, the data from Bill's students show that there is a relationship between the degree of importance and the degree to which students like mathematics in school for most activities. This means that those students who consider an activity as important also indicated that they liked it. However, similar to Alice's students, the importance of mathematics is rated higher than the degree to which they liked it.

The second dimension of the ATMT deals with Mathematics as a Process. The responses suggest that students view trigonometry as a set of rules. These rules are used in solving problems and, in fact, the problems cannot be solved without the use of these rules. This view about trigonometry has likely been shaped by Bill's emphasis on developing formulas and theory and exploring the reason behind these. However, Bill's students do believe that trigonometry does teach them to think logically. Given Bill's teaching style, that incorporates a problem-solving approach and a search for multiple solution paths, students have come to believe that trigonometry is a good unit of study for creative people.

With respect to Mathematics and the Student, the data suggest that students want to do well in trigonometry, they feel good when solving a mathematics problem by themselves, they feel challenged when given a difficult mathematics problem and they think mathematics is fun. The overall responses suggest that Bill's students have a positive attitude toward mathematics.

The attained curriculum as measured by the MDE indicates that Bill's students performed slightly below (62%) the provincial average (64%) for trigonometry. The achievement results for Math 30 (62%) are also slightly below the provincial average (63%). While it is recognized that there are a number of factors that influence achievement results, Bill's classroom practices seem to be a contributing factor. In particular, his strong process orientation and problem-solving emphasis are not congruent with the nature of the MDE, which is for the most part a skill oriented test. Bill is quite aware that teaching for the MDE is different in nature from teaching for success at the university.

Bill's classroom practices seem to reflect an emphasis on the latter.

Carl: The daily classroom routine is well established and it is best described as emphasizing teacher remedial explanations, directive teaching and student practice. During the remedial explanation phase Carl usually responds to questions from previous assignments, activities or homework. Invariably students would raise many questions dealing with content from lessons that were taught several days ago. For example, during one such lesson three students asked questions about quadrants, reading trigonometry tables, changing degrees to radians and finding values of reciprocal trigonometric functions on a scientific calculator which only has primary trigonometric functions. Carl willingly answered the questions posed by these three students which took 23 minutes out of the 40 minute period. The other seven students quietly worked on exercises related to the most current lesson. These remedial explanations are almost a daily occurrence and on most days these exceed well over 50% of the class period, leaving Carl with relatively little time for lesson development and even less time for student practice. However, checking an answer to a problem by going over it is viewed as the most important activity by Carl's students. This practice has the effect that after each lesson students are left with many unanswered questions which are then presented to Carl during the next mathematics class. While the elements of Carl's classroom routine are similar to those of Alice and Bill, the major difference is in his emphasis on the mechanical treatment of skills as compared to the development of concepts using a problem-solving approach. The teaching strategies used most frequently by Carl involve teacher demonstrations, class discussions and drill and practice. Other strategies used less frequently include student-directed class discussions, having students put homework on the blackboard and checking homework where Carl solves all the problems on the blackboard. Problem-solving and applications, worksheets, tests and quizzes are only included on one or two occasions during the trigonometry unit. Small group activities are never used as part of a plan, but students are allowed to work together during the student practice phase of the

lesson. Carl's problem-solving and application activity is limited to the "word problems" contained in the text and it is a computational activity in isolation from a context of "real-life" applications. Problem-solving is still a separate activity for Carl for which he requires complete mastery of skills before students participate in it. Carl has a total of 30 hours allocated for trigonometry which is used to teach the core curriculum. His treatment of core topics is strongly influenced by the MDE. The use of previously written MDE is not just for review and practice purposes, but they also guide him in determining the depth and breadth of his lessons. In some instances, Carl extends core topics to make it interesting or challenging for some students and in other instances he omits content because of student ability or it is assumed not to be on the MDE. The reported use of the 30 hour time allocation for Carl is as follows: developing and applying the sine and/or cosine law (6 hours), reviewing and maintaining previously learned materials (5 hours), solving equations involving trigonometric functions (5 hours), trigonometric identities (4 hours), applications of trigonometric functions (3 hours), dealing with the unit circle (2 hours), trigonometric functions as ratios of the length of sides of a right triangle (2 hours), graphing of trigonometric functions (1 hour), finding domain and range of the six trigonometric functions (1 hour) and developing the concept of periodicity (1 hour). Carl makes no provision for developing elective topics and the concept of a wrapping function. Although Carl exceeds the recommended time allocation for trigonometry, he is still identifying a serious time concern. As a result, his teaching is closely aligned with pedagogical efficiency, the use of limited textual resources, an emphasis on a product orientation and limited provision for student evaluation. The evidence seems to suggest that Carl's time problems are more a function of his use of the forty minute class period than a real shortage of time. The students in Carl's class spent the major portion of their class time listening to his explanations and expository teaching. Only a small portion of the class period is used for student practice. As a result of the time problem, Carl's students are fairly infrequently subjected to tests, quizzes and worksheets. His emphasis remains on presenting core content and making the students paper-and-pencil proficient. Carl's students do use calculators, but computers are not used at all.

With respect to homework, Carl assigns it minimally and only when it is important to reinforce a particular skill or concept. However, Carl does expect students to do homework every night in areas that students feel they have the greatest need. If students have questions or they experience problems, he is quite prepared to provide individual assistance in class. Carl feels that his small enrollment of ten students allows him to provide a high level of individual help.

The use of instructional resources in Carl's classroom is essentially limited to one text which is one of the prescribed references and the curriculum guide. Carl's core program is to a large extent defined by the text. There is only occasional reference to a text from the previous curriculum and the use of materials prepared by himself. The resource base used by Carl is considerably narrower than that used by Alice or Bill.

Carl tries to maintain a strong focus on students' interests and abilities in his teaching. He has known his students for several years which allows him to respond to individual needs more effectively.

Given Carl's interpretation of the intended curriculum, the attained curriculum as measured by the ATMT suggests that his students' attitude toward Mathematics in School is a reflection of his own classroom practice. For example, Carl invested most of his own class time in checking answers to problems by going over these with the students and consequently students felt that this is an important activity. Students also placed considerable importance on using algebraic techniques for dealing with trigonometric problems. This seems to be an indication that students recognize their own lack of skills and understanding related to algebraic manipulations. Classroom observations confirmed that on numerous occasions students had difficulties with the algebraic treatment of trigonometric expressions (eg. $1/\cos \theta + 1/\sin \theta$). The importance that students attach to memorizing trigonometric formulas and theorems is a direct reflection of Carl's classroom practice and his view of mathematics. Carl has a rule or skill-oriented view of mathematics which employs a set of mechanical procedures to be memorized and applied when solving problems. His emphasis on recalling facts, mastering basic skills and

producing the correct solution seems to account for the fact that students attach such high importance to checking answers to problems. In general, there seems to be no relationship between what students feel is important with what they say they like. For example, students attach a high degree of importance to memorizing trigonometric formulas and theorems, but they describe it as the least liked activity. Another case in point is the activity of using electronic devices for solving problems in trigonometry. While students felt that this activity is relatively unimportant, they indicated that they liked it the most. Overall, Carl's students rated the importance of Mathematics in School higher than the degree to which they liked it.

With respect to the students' attitude toward Mathematics as a Process, Carl's students suggest that not only is there always a rule to follow in solving trigonometric problems, but these problems cannot be solved without using a rule. Furthermore, they feel that trigonometry helps one to think according to strict rules, but it also helps one to think logically. The classroom practices used by Carl seem to be clearly reflected in the students' attitudes.

The third dimension of the ATMT deals with Mathematics and the Student. Carl's students responded quite positively toward this dimension. Students indicated that they feel good when solving a mathematics problem by themselves, that they really want to do well in trigonometry and that they feel challenged when given a difficult mathematics problem.

The attained curriculum, as measured by the MDE, shows that Carl's class performed significantly below (36%) the provincial average (61%) for trigonometry. As well, the overall achievement results of his class for Math 30 (51%) were also significantly below the provincial average (65%). Based on the classroom observations, there appear to be a number of factors that affected these results. Carl consciously aligned his skill-oriented classroom practices with the nature of the MDE. However, his narrow focus on teaching his students one particular skill and solution path, seems to be responsible for his students' poor achievement results. Even the skills that were covered in Carl's class were often poorly understood by students as the student results from quizzes and tests, as well as from the student questions seem to suggest. Equipping

students with one solution method creates difficulties for the students, particularly when the problem that is posed on a test is slightly different from what the student has learned. Carl was also strongly influenced by a number of situational factors (eg. forty minute class periods and teaching seven different courses) that affected his effectiveness as a teacher.

C. Influencing Factors

The focus of this question is on identifying and describing some of the factors that influence the translation of curriculum material into classroom practice.

Question 3: What are some of the factors which influence the teachers' interpretation of the trigonometry unit to students?

The findings from the current study suggest that there are factors associated with two major areas of influence:

1. Factors associated with the four curriculum commonplaces.
2. Teacher factors.

In Chapter VII it is illustrated how the teachers' functional paradigm reflects the various factors of influence.

Factors Associated With the Four Curriculum Commonplaces

The factors identified under the four curriculum commonplaces for each case suggest that there is some variation among mathematics teachers. The factors for each teacher are identified in the case reports of Chapter IV and in TABLE 3 (Chapter VII). The findings for the current study are based on classroom observations, situational interviews, teacher interviews and the CPTT survey. It is assumed in this study that the meaning that curriculum materials have for teachers can be inferred from what they say about their use of the

materials and from the observable behavior of teachers. Furthermore, it is assumed that from these meanings and observations valid inferences can be made about the teachers' beliefs and values and the influence they have on interpreting curriculum materials.

The use of Schwab's four curriculum commonplaces is not to suggest that there are four distinct categories in the minds of teachers, but rather, they assist in guiding the discussion. The current study, as well as other related research, (Connelly & Elbaz, 1980; Crocker, 1979) suggests that the elements of the curriculum commonplaces are closely interwoven and that the perceptions under one are not mutually exclusive of the other three commonplaces. In some cases teachers hold views somewhere between the extremes of a particular perception continuum.

Perception of School Setting: Under this category three related perceptions were identified. The first perception focuses on school facilities, which identified a lack of equipment and resources resulting in a restriction of student activities. The second perception focused on the expectations of the community particularly as it relates to student achievement. As a result of these community expectations, many teachers have adopted a product oriented approach with regard to curriculum materials and teaching strategies. Student achievement is a significant factor in teacher interpretation of curriculum materials since the introduction of mandatory province-wide mathematics diploma examinations and all other academic grade twelve subjects in Alberta. The third perception focused on situational factors which includes the school resources, number of mathematics teachers, school timetable, teaching assignment (ie. number of students and subjects) and the nature of the students. The teachers suggest that all of these situational factors either have a limiting or enhancing effect on the quality of materials prepared, the teaching strategies and evaluation methods used, and the teachers' attention to individual student needs. The data collected in this study indicate that teachers hold a range of beliefs and values about each of the four curriculum commonplaces. In general, the data suggest that the perceptions of mathematics and teaching seem to dominate

teachers' interpretation of curriculum materials.

Perception of School Mathematics: In this study, seven related perceptions of Math 30 were identified. One view is that Math 30 is limited to topics listed in the core curriculum. This view is particularly associated with those teachers who use the total time allotment to teach core topics because elective topics are not part of the MDE. Another view is that Math 30 includes a more personal set of topics outside the core curriculum. Although these topics differ for each teacher, there are, however, several commonalities (eg. formal proof of trigonometric identities, the use and proof of half angle formulas). In some cases these personal topics are merely "pet lessons" that go well beyond the curriculum objectives. In other cases, teachers have not yet "let go" of topics from a previous course or curriculum.

The third perception of Math 30 is the need to master basic skills. This perception is grounded in the belief that an understanding of the mathematical structure can only be achieved when the basic skills are mastered. This view of school mathematics is particularly held by teachers who subscribe to a product orientation in their teaching.

The perception of Math 30 that emphasizes major theoretical concepts and principles is primarily associated with a de-emphasis of applied or practical mathematics in the curriculum. The elective component is often used to study topics dealing with major theoretical concepts and principles. This emphasis is justified by teachers who believe that students will need this kind of background to succeed in post-secondary institutions.

A fifth perception of Math 30 focuses on mathematics as a unified whole. This perception is associated with teachers who value understanding the logical connection of each part to the overall structure of mathematics. These teachers typically review content and concepts by constantly relating the current material to previously taught material. A sixth perception of Math 30 focuses on mathematics as a problem solving tool. Teachers associated with this perception have a tendency to view mathematics and problem solving as synonymous.

A seventh perception of Math 30 focuses on the nature of mathematics,

and it is associated with teachers who use mathematical content to illustrate the processes and who teach inductive and deductive reasoning, the nature of proof, and the power and philosophy of mathematics.

Perception of Teaching: The data collected in this study suggest eight typical perceptions of teaching. One perception places a high value on student understanding and success on the MDE. This understanding refers specifically to the content included on the MDE. This view is again closely related to the findings by MacDonald and Leithwood (1982) who state that a major factor in curriculum decision making is the teachers' overriding need for "achievement". A second perception places a high value on pedagogical efficiency, that is, a time effective means of giving students a clear "understanding" of mathematical concepts and principles. The data in this study suggest that for some teachers student understanding is indicated by the student being able to correctly answer assigned questions. This perception is associated with teachers who select materials and tests on the basis of how quickly they can be completed, evaluated and returned to the student. Quizzes and worksheets are often chosen by teachers who value pedagogical efficiency. The fact that most teachers experience a sense of being rushed probably contributes to the valuing of pedagogical efficiency. Also, teachers who teach several different courses and have large classes seem to hold this value the strongest. It is not uncommon for teachers to limit the content and omit content or activities in the name of pedagogical efficiency. Teachers who value pedagogical efficiency also want curriculum materials to provide an abundant supply of student exercises. Another view of teaching mathematics emphasizes academic rigor. This view does not preclude an appreciation of pedagogical efficiency, but it is associated with those teachers who consider the mathematics curriculum as an absolute minimum to be covered. These teachers supplement extensively with additional materials, challenging problems and activities aimed at developing students to their "full potential". The emphasis on academic rigor seems to be connected with the view that Math 30 is a select academic course intended for university-bound students.

A fourth perception is that an essential component of effective teaching involves motivating students. This normally involves a variation of class activities and the presentation of interesting challenges for students. Teachers associated with this view normally accept the fact that Math 30 students will have wide ranging interests and abilities and not everyone will be university bound.

The value placed on high and clear expectations of students is normally associated with those teachers who value academic rigor. Teachers associated with this view tend to be somewhat inflexible in their demands and they would likely not consider program modifications to meet the needs of particular students. This view frequently serves in the course selection process by students.

Valuing mathematics as a process is another perception held by many teachers. Teachers associated with this view emphasize exploratory activities with an emphasis on problem solving. They are completely comfortable "loosing themselves" with the students in a mathematical problem, trying different solution strategies, looking back, posing new problems and finally arriving at a solution. Their teaching is completely process oriented and finding the solution to a problem is secondary.

A seventh perception of teaching mathematics is associated with teachers who value feedback on student performance. These teachers have a strong need to know through informal and formal means what the level of student understanding is. This is primarily done through quizzes and tests, worksheets, homework and verbal interaction. The reasons for doing this is not always the same for all teachers. Some use this information for further lesson planning or review, others use these evaluative devices to motivate students or to impose academic rigor and still others want to solidify their information base for parents.

The eight perception of teaching mathematics is associated with teachers who value professional competence. This view is fairly prevalent among all teachers. The data suggest that teachers are prepared to limit treatment of topics or discussion to the teacher's own competence or comfort level. Other

teachers select activities that worked well for them in the past or that are based on their personal interest and background.

Perception of Students: Three perceptions of students emerged from the current study, one focusing on students' interest, the other on students' abilities and the third on students' success in future careers.

The perception which focuses on student interest is associated with choosing topics and activities that students will enjoy. The data suggest that teachers mention student interest quite frequently as the reason for selecting particular topics, approaches or activities. The strong focus on student interest is associated with those teachers who want to teach more than just content. These teachers want their students to have a genuine interest in and liking for mathematics. As well they have a strong need of positive affiliation with students.

The focus on students' ability is associated with selecting curriculum materials on the basis of what is considered appropriate for the students' level of ability, while rejecting that which is considered to be too easy or too difficult. Some teachers make few program modifications or changes in approach because they choose to assist students who are having difficulty on a one-to-one basis either within or outside the class.

The perception of students which focuses on students' success in future careers is associated with those teachers who want to ensure that their students have the proper academic standing and prerequisite skills for further study. These teachers will also continually stress the importance of being successful on the MDE. Many teachers take it quite personally when students fail to achieve the required academic standard at school that allows them to enter the institution of their choice.

Teacher Factors as Influences on Curriculum Interpretation

From the interviews with and observations of teachers it is evident that there are teacher factors that constitute an important influence on the teacher's

interpretation of curriculum materials. The factors that most influence teachers are their training in mathematics (ie. number of mathematics and methods courses) and their experience in teaching mathematics. During several interviews all three teachers alluded to their training and experience, indicating that their extensive mathematics background and mathematics teaching experience gave them a strong sense of security. Alice expressed some uncertainty with her interpretation of the curriculum which she related directly to her brief mathematics teaching experience. Those teachers who had extensive experience teaching mathematics had an extensive reservoir of teacher-prepared materials. They also saw themselves better prepared to incorporate mathematical problem solving and applications as compared to a teacher with less experience. While there are other factors at play, the data suggest that as teachers gain more training and experience, they become more independent and rely less on prescribed curriculum materials. Alice had a far greater need to interact with other teachers for assistance and clarification than Bill and Carl who had significantly more experience in teaching mathematics.

Summary: The study suggests that the various values, beliefs and perceptions that teachers hold are reflected in their functional paradigm. In addition, it reflects their specific teaching situation and teacher factors. The resulting classroom practices as manifested in various techniques, exemplars and routines become a well established decision-making system referred to as the teachers' functional paradigm.

VI. TEACHER COMMONALITIES

This chapter contains three major sections. The first section presents a cross-case analysis, which focuses on comparing and contrasting three interpretations. A cross-sample analysis which focuses on situating the three cases within the context of a small sample of mathematics teachers from across the province is presented in the second section. The third section presents the answer to the fourth major research question which deals with the relationships among the influencing factors.

A. Three Interpretations: Comparisons and Contrasts

The purpose of this comparison is to compare and contrast the three teachers in relation to the four curriculum commonplaces with the intention of identifying common elements of their interpretation. This comparison serves as an information base for answering the fourth major research question in the third section of this chapter and for discussing the nature of the teachers' functional paradigm in chapter VII.

A comparison of the three cases suggests that there are a number of common perceptions as well as differences in perceptions with respect to school setting, school mathematics, teaching and students. The school setting factor has an important impact on all three teachers. Class size, number of mathematics teachers, and the level and quality of interaction are common factors among all three teachers. In addition, Alice identifies the availability of resources, Bill lists school size and staff size and Carl identifies teaching load, length of class period, ten month versus semester structure, number of different courses taught, number of students taught, student busing and the nature of the students as additional school setting factors that influence the interpretation of curriculum materials in the classroom. Alice's belief in the use of technology is also shared by Bill and Carl, except that the belief by the latter two is restricted to calculators. While Alice uses calculators and computers in the classroom, neither Bill nor Carl use them because of their lack of skill and background with

them. The feelings with respect to having a good text are also shared by all three teachers. The prescribed text for the course presents considerable frustration to the teachers because of the inadequate coverage of topics, the many mistakes and the low quality as a problem-solving resource. The expectations of the community are seen as another strong influence on all three teachers. There are expectations in all three communities that the students do well in the course and on the MDE. The case teachers feel that a high failure rate would be seen as a reflection on their teaching. As a result, the classroom practices of all three teachers have changed and continue to change in order to keep the failure rates to a minimum.

Alice sees school mathematics as a well established body of knowledge, concepts, skills and procedures whose content is sequentially arranged and whose parts are logically connected. In her view, there is a definite set of topics from the body of mathematical knowledge which ought to be included in the trigonometry unit. While the content of the unit is largely what it should be, she feels that the inclusion of half and double angle formulas should be accommodated in the unit. In fact, Alice includes these as part of her core curriculum. These inclusions are quite consistent with her belief in core topic extensions particularly if they are useful for further study in mathematics or if they are interesting examples of mathematical reasoning.

Bill seems to share Alice's view of school mathematics with the addition that this body of knowledge, concepts, skills and procedures must be explored by students. For Bill as well, the topics in trigonometry could also be extended to include more work with trigonometric formulas, formal proofs and mathematical applications. Bill in particular is still incorporating materials from the previous curriculum in the present curriculum as is evidenced by his frequent reference to a text that was authorized for the previous curriculum. Bill readily admits that he is partial toward the previous text. He feels that the mathematical rigor of the new curriculum and text have been significantly reduced.

Alice's adoption of the new curriculum is more complete than Bill's because she was more active at the system and provincial level during the implementation

process of this curriculum. This interpretation would be consistent with Fullan's (1982) contention that personal contact or experience is the primary determinant of teacher acceptance of curriculum innovations.

Carl perceives school mathematics as a set of topics that contain skills, rules, procedures and principles to be memorized and applied when solving problems. This view manifests itself clearly in his isolated treatment of topics and his belief in the importance of students being able to recall facts, produce correct solutions and master basic skills. Carl attributes this view of school mathematics to the existence of the MDE which has strongly influenced his teaching to a point of being completely product oriented. Carl is even advocating a further reduction of theoretical concepts and he has confined the core content to what he assumes to be on the MDE.

Carl's view of school mathematics fits only in a small way the interpretation of Alice and Bill. That is to say that Carl is positioned at different locations along the dimensions that make up the interpretation domain. Whereas Alice and Bill view school mathematics as a body of knowledge, concepts, skills and procedures whose parts are logically connected, Carl views it as a set of topics that are separate and apart. All three teachers limit their program to the core curriculum with occasional extensions of core topics being provided by Alice and Bill and minimally so by Carl. The findings also suggest while Carl believes in the separate treatment of topics, Alice and Bill believe in establishing connections among mathematical topics. For example, Alice would briefly review the elements of the Cartesian plane before she commences with her lesson on graphing or she would review the concept of an irrational number as the class discusses coordinate points on the unit circle. Bill would engage in drawing similar connections between current experiences with a concept and the students' previous experiences with that same concept. For example, Bill would not take for granted that students recognize that the distance from the origin to a point on the terminal arm of an angle θ is calculated using the distance formula, which is a term students have learned in the previous course. Carl, however, would seldom make such connections among mathematical topics. However, with regard to the elective component,

all three teachers omit it from their mathematics program.

With regard to their perceptions on teaching, all three teachers follow a well-established routine consisting of four types of activities: review, lesson development, teacher demonstrations and student practice. Furthermore, they value pedagogical efficiency. Selecting efficient teaching techniques which allow them to make the best use of the time available for teaching mathematics is one example that demonstrates this value. Pedagogical efficiency is also evident in evaluating students, test administration and determining depth of program coverage. Other values shared across the three cases include: student understanding and success on the MDE, student expectations and professional competence. The frequent reviews and practices, using previously written MDE, are also common across the three cases. Although the impact of the MDE is felt the most by Carl, the other two teachers allude to its influence as well.

For Carl, the expository teaching style with a focus on basic skills and mechanical treatment of concepts and problems seems to be the preferred style. Alice prefers teacher demonstrations, class discussions and problem solving activities. Bill is the farthest along the problem-solving continuum. All of his actions and activities suggest that he values mathematics as a process and emphasizes problem solving and applications as a way of teaching.

Alice shares with Bill and Carl the belief that students have a good understanding of the trigonometry unit. Understanding here means that students receive passing marks on the various teacher evaluation instruments. All of them are prepared to assist their students individually and only Carl confines his assistance to 'in class'. While all three teachers recognize the wide range of abilities among their students, Alice seems to be most accepting of that. Bill and Carl are somewhat concerned with inappropriate placements of students in their class, but both are making appropriate provisions to meet their needs. The focus on the students' interests, abilities and future careers is shared by all three teachers. The relationship between the teachers and their students is a friendly one in all three cases.

Alice believes that her students have a very good attitude toward

learning mathematics. The results of the ATMT for Alice's class do not entirely confirm this. The degree to which students like mathematics in school is only minimally related to the degree of importance that they attach to it. This means that activities that are seen as important by students are not necessarily liked by them. The converse is true as well. In general, students rated the importance of mathematics in school higher than the degree to which they liked it. Student responses to Mathematics as a Process seem to suggest that trigonometry is a set of rules that can be followed when solving problems. Alice's students feel that it helps one to think logically and in accordance with strict rules and there is always a different way to solve most trigonometric problems. The responses to the third dimension of the ATMT, Mathematics and the Student, suggest that the students feel good when they solve a mathematics problem by themselves, that they really want to do well in trigonometry and that they usually understand what is being talked about in trigonometry. Bill, too, believes that his students have a good attitude toward learning mathematics. The attitudes of Bill's students are generally more positive than those of Alice and Carl. Student responses to Mathematics in School suggest that those students who consider an activity as important also indicated that they liked it. The overall student responses show that the importance of mathematics in school is slightly higher than the degree to which they liked it. With regard to Mathematics as a Process, Bill's students view trigonometry as a set of rules, but they also feel that it helps one to think logically. The responses to Mathematics and the Student suggest that students want to do well in trigonometry, they feel good when solving a mathematics problem by themselves and they feel challenged when given a difficult mathematics problem.

The attitude as expressed on the ATMT is similar for Carl's students as for Alice's and Bill's students. The overall results on Mathematics in School indicate as well that students rate the importance of mathematics in school higher than the degree to which they like it. The student responses to Mathematics as a Process suggest that not only is there always a rule to follow in solving trigonometric problems, but these problems cannot be solved without using a rule. Carl's students feel that trigonometry helps one to think according

to strict rules, but it also helps one to think logically. His students express fairly positive feelings on the dimension dealing with Mathematics and the Student. In particular, they feel good when solving a mathematical problem by themselves and they want to do well in trigonometry.

B. Situating the Cases

The focus of this section is on, "How do the individual cases (referred to as case teachers) compare with the provincial sample of six teachers (referred to as sample teachers) in relation to the major factors of influence on curriculum interpretation?" It is recognized that a sample of six mathematics teachers is not adequate to represent the mathematics teacher population of the province, but it allows the three cases to be situated within the context of a small sample of mathematics teachers teaching the same mathematical topic.

The third major research question in Chapter 5 identifies two significant areas of influence on the interpretation of curriculum materials. These two areas are teacher factors and factors associated with the four curriculum commonplaces. TABLE 2 presents a summary of the responses to the CPTT for the sample teachers and the three case teachers. The teacher factors are made up of six elements and the four curriculum commonplaces: school mathematics, teaching, students and school setting; consisting of seven, five, three and nine elements, respectively. In order to facilitate discussion of the data, the findings are categorized into five sections which deal with the teacher factors and Schwab's four curriculum commonplaces.

TABLE 2

**SUMMARY OF COMPARISON OF CASE TEACHERS WITH
SAMPLE TEACHERS IN RELATION TO MAJOR FACTORS
OF INFLUENCE ON CURRICULUM INTERPRETATION**

MAJOR FACTORS	SAMPLE TEACHERS	ALICE	BILL	CARL
A. TEACHER FACTORS				
1. TRAINING (MATH & METH)	- 3.5-7.5+ math courses - <1 to 3 meth courses	- 7 math courses - 2 meth courses	- 6 math courses - 2 meth courses	- 7 math courses - 2 meth courses
2. EXPER. TEACHING MATH	- 6-14+ years	- 2 years	- 11 years	- 20 years
3. ADEQUACY TO TEACH ELECT.	- satisf. to v. good	- good	- very good	- very good
4. ADEQUACY TO TEACH P.SOLV	- unsatisf. to v. good	- satisfactory	- very good	- very good
5. ADEQUACY TO TEACH APPL.	- unsatisf. to v. good	- satisfactory	- very good	- very good
6. ADEQUACY TO IMPLEMENT: OVERALL PHILOSOPHY, GOALS AND OBJECTIVES:	- satisfactory to v. good	- good	- very good	- very good

B. CUR. COMMONPLACES

a. SCHOOL MATHEMATICS

1. TRIG. SUBTOPICS

TREATMENT:

- full range
- same as sample teachers.
- 3 types: (1) taught them (2) - 3 types: (1) taught them (2) reviewed and extended and (3) reviewed and extended and (3) assumed as prerequisite not covered. knowledge.

REASON FOR TREATMENT:

- (1) They were included in cur, (2) they were useful for further study in math and (3) they were assumed to be on the MDE.
- (1) They were included in the cur and (2) they were useful for further study in math.
- same as sample teachers.
- same as sample teachers.
- when subtopics were the same as cur objectives they were either taught, reviewed and extended or reviewed only.
- not found
- not found
- not found

2. TRIG OBJECTIVE EMPHASIS

HEAVY EMPHASIS ON:

- (1) define radian measure, (2) solve oblique triangles by using the sine and cosine law, (3) apply sine and cosine law to practical problems, (4) solve simple trig equations and (5) given value of one trig ratio, evaluate the other five trig ratios.
- (1) define the unit circle $x^2 + y^2 = 1$, (2) determine coordinates of points on unit circle, (3) define trig ratios in terms of coordinates of points on the unit circle.
- (1) maintain previously developed skills, (2) solve trig equations, (3) derive and apply quotient, reciprocal and Pythagorean relation and (4) solve oblique triangles by using sine and cosine law.

3. TRIGONOMETRIC FORMULAS:

TREATMENT:

- trig formulas were either formally proven or stated with informal justification.
- formally proven, stated with informal justification, stated with informal justification, stated without justification and some not covered.
- formally proven, stated with informal justification or some justification or formally proven. not covered.
- not covered, stated without justification or formally proven.

REASONS:

- proof unifies basic ideas and is an interesting example of math reasoning.
- same sample teachers
- proof unifies basic ideas.
- proof unifies basic ideas.

EXPECTATIONS:

- students were expected to provide: (1) informal justification of formula and apply it and (2) recall formula and apply it.
- (1) prove and apply formula, (2) recall and apply it and (3) apply formula when given.
- (1) informal justification and apply and (2) prove and apply. informal justification and apply.
- (1) no justification, (2) recall and apply formula and (3) informal justification and apply.

4. TRIGONOMETRIC INTERPRETATION

USED:

- used ratio and unit circle reference interpretation.
- same with minimal use of wrapping function.
- ratio, unit circle reference interpretation and line representation. Minimal use of wrapping function.
- same as sample teachers.

REASON FOR USE:

- (1) in curriculum, (2) easy & enjoyable for students and (3) expected on MDE.
- (1) relates closely to math studied in prior grades and (2) it is easy for students to understand.
- (1) relates closely to math of prior grades, (2) easy for students to understand and (3) is presented in text.
- expected on the MDE.

REASON FOR NON-USE:

- wrapping function and line representation not included because too difficult for students and teachers were unfamiliar with them.

- wrapping function and line representation not included because it was not in the text nor emphasized in the curriculum guide.

5. SKILL EMPHASIS

PRIMARY IMPORTANCE:

- (1) skill in solving for missing parts of right triangles and (2) skill in applying trigonometry to application problems.

- (1) skill in solving for missing part of any triangle, (2) skill in proving trig identities, (3) skill in algebraic techniques for dealing with trig problems, (4) skill in solving trig equations, (5) skill in graphing trig functions and (6) skill in use of electronic devices for solving trig problems.

- (1) skill in solving for missing parts of right triangles, (2) skill in solving for missing parts of any triangle, (3) skill in applying trig to application problems and (4) skill in using electronic devices for solving trig problems.

6. CURRICULUM MATERIAL SELECTION

INFLUENCED BY:

- (1) what students will need to know in order to do well on the MDE, (2) the guidelines in the curriculum guide, (3) the students ability, (4) time available, (5) approaches that have worked well in the past and (6) what students will need to know in order to succeed in their post-secondary education.

- (1) what students will need to know in order to succeed in their post-secondary education, (2) what students will need to know on the MDE and (3) what is expected in the curriculum guide.

- (1) what students require in order to understand the structure of math, (2) the guidelines in the curriculum guide, (3) and approaches that have worked well in the past.

- (1) what students will need to know in order to succeed in their post-secondary education and (2) what students will need to know in order to do well on the MDE.

7. TIME ALLOCATION

MOST TIME SPENT ON:

- (1) unit circle, (2) developing and applying law of sine and cosine, (3) trig identities, (4) application of trig functions and (5) solving trig equations.
- (1) testing and follow up, (2) trig identities, (3) developing and applying sine and cosine law, (4) graphing trig functions and (5) solving trig equations.
- (1) developing and applying sine and cosine law, (2) trig identities, (3) graphing trig functions, (4) concept of wrapping function, (5) application of trig function and identities and (6) solving trig equations.
- (1) developing and applying sine and cosine law, (2) reviewing and maintaining previously learned materials, (3) solving trig equations, (4) trig identities and (5) applications of trig functions.

AMOUNT OF TIME:

- range of 25 - 35 hours, mean of 29 - 25 hours
- 40 hours
- 30 hours

b. TEACHING

1. VALUES HELD:

- student understanding and their success on MDE.
- same as sample teachers.
- same as sample teachers.

2. TREATMENT OF TRIG FORMULAS:

- exceeds curriculum expectations for most trig formulas.
- same as sample teachers.
- same as sample teachers.
- fell below curriculum expectations for most trig formulas.

3. TEACHING STRATEGIES:

- (1) lectures, (2) teacher demonstrations, (3) class discussion and (4) drill and practice.
- (1) teacher demonstrations, (2) class discussions, (3) drill and practice and (4) checking homework with teacher solving assigned problems on the blackboard.
- (1) teacher demonstrations, (2) drill and practice and (3) checking homework with teacher solving assigned problems on the blackboard.
- (1) teacher demonstrations, (2) class discussions and (3) drill and practice.

4. RESOURCE MATERIAL USED:

- (1) Math Is/6, (2) teacher-prepared material and (3) Curriculum Guide.

- (1) Math Is/6, (2) Foundations of Math for Tomorrow: Senior, (3) Curriculum Guide and (4) teacher-prepared material.

- (1) Math Is/6, (2) Modern Intermediate Algebra and (3) teacher-prepared material.

- (1) Math Is/6 and (2) Curriculum Guide.

5. FACTORS AFFECTING SELECTION OF CURRICULUM MATERIALS:

- (1) what students need to know in order to do well on the MDE, (2) Math 30 Curriculum Guide, (3) students ability, (4) time available, (5) approaches that worked well in the past and (6) material that builds on or extends concepts covered in previous grades.

- (1) what students will need to know in order to succeed in their post-secondary education, (2) what students will need to know in order to do well on the MDE, (3) Math 30 Curriculum Guide, (4) students ability, (5) time available and (6) approaches that have worked well in the past.

- (1) what students require in order to understand the structure of math, (2) Math 30 Curriculum Guide and (3) approaches that have worked well in the past.

- (1) what students will need to know in order to succeed in their post-secondary education and (2) what students will need to know in order to do well on the MDE.

c. STUDENTS

1. ATTITUDE OF STUDENTS TOWARD MATHEMATICS:

- satisfactory to good

- very good

- good

- good

2. UNDERSTANDING OF TRIGONOMETRY:

- satisfactory to very good

- good

- good

- good

3. RELATIONSHIP FOCUS OF TEACHERS:

- on students interests and abilities

- same, plus focus on their future careers

- same, plus focus on their future careers

d. SCHOOL SETTING

1. NUMBER OF COURSES CURRENTLY TEACHING:	- 4 or more courses	- 3 courses	- 2 courses	- 7 courses
2. NUMBER OF STUDENTS CURRENTLY TEACHING:	- 50-150 different students	- 78 diff. students	- 51 diff. students	- 124 diff. students
3. PERCENTAGE OF MATH TEACHING:	- 60-100% mathematics	- 100% mathematics	- 100% mathematics	- 29% mathematics
4. PHYSICAL SETTING:	- rated satisfactory to good	- good	- very good	- good
5. GRADE ARRANGEMENT AT SCHOOL:	- K-12, 7-12, 9-12, 10-12	- 10-12	- 8-12	- K-12
6. NUMBER OF STAFF:	- ranged from <20 to <60	- 26	- 15.5	- 19
7. NUMBER OF MATH TEACHERS:	- ranged from 1-8	- 4	- 5	- 9, incl. elementary
8. STUDENT ENROLLMENT:	- ranged from <500 to 999	- 450	- 225	- 325
9. USE OF TECHNOLOGY:	- occasional use of computers by one teacher	- calculator use emphasized	- calculator use emphasized	- calculator use emphasized
		- uses computers for graphing trig functions	- computers not used	- computers not used

Teacher Factors : The information in TABLE 2 indicates that the level of training and mathematics teaching experience for case teachers is strikingly similar to that of sample teachers. The case teachers, however, rated their adequacy to teach elective units, problem solving and applications slightly higher than some of the sample teachers. That slight edge of the case teachers over some of the sample teachers is also reported with respect to their adequacy to implement the overall philosophy, goals and objectives of the program. Since the case teacher find that their background is a significant factor in affecting the translation of curriculum materials for use in the classroom, it is reasonable to assume that sample teachers are affected by that factor as well.

The summary of the teacher factors suggests that in the context of this small sample, the case teachers are fairly representative of the sample teachers.

School Mathematics: The responses of the sample teachers to the Trigonometric Subtopic Questionnaire (TSQ) suggest that the subtopics received the full range of treatments. However, when a subtopic listed was the same as one of the stated curriculum objectives of the trigonometry unit, the sample teachers either taught it, reviewed and extended it or reviewed it only. The primary reasons for teaching or reviewing these subtopics were: (1) it is included in the Math 10-20-30 Curriculum Guide (1983), (2) it is useful for further study in mathematics, and (3) it is assumed to be on the MDE. For the case teachers as well, the subtopics received nearly the full range of treatments and for those that were either taught or reviewed, the reasons cited were almost identical to those stated by the sample teachers. In instances where subtopics were not taught, the reasons stated for not teaching them were simply the opposite of the reasons cited above.

The sample teachers seem to give all trigonometry objectives medium to heavy emphasis, but there is a considerable range of emphasis on several objectives. The findings are also similar for the case teachers. In some instances the responses to a particular objective are on both ends of the

response scale. The responses for the case teachers range from low emphasis to heavy emphasis with close agreement on several objectives. From the observations of case teachers it is evident that such extreme differences in emphasis are not uncommon among teachers. These differences are the result of conscious choices on the part of the teachers. While there are many reasons for making these choices, the main reasons that this study identified are : teachers' misinterpretations of the Curriculum Guide, teachers' assumptions that certain content is on the MDE, teachers' judgments about their students' ability, teachers' concern about time, teachers' success with a certain topic in the past, teachers' own interest, ability and background, teachers' own view of what is important and teachers' "pet topics".

With respect to trigonometric formulas most sample teachers covered the formulas either by formally proving them or stating them with informal justification. The primary reason for providing a formal proof was that the proof unifies basic ideas, whereas the second reason was that the proof is an interesting example of mathematical reasoning. With regard to the teacher's expectations of student mastery for the various formulas, sample teachers covered the full range, but in general converged on two expectations: that students are capable of providing an informal justification of the formula and apply it and that students can simply recall and apply the formula. There were considerable differences among the case teachers and between the case and sample teachers particularly as it relates to the treatment of these formulas and the expectations of students. Carl did not cover most of the formulas or discuss them with the class, whereas Bill or Alice would give a formal proof for the same formulas. Similar differences exist with respect to teachers' expectations of student mastery for the various formulas. Carl does not expect students to justify formulas. He simply expects students to recall formulas and apply them, whereas Alice and Bill have expectations that students prove and apply certain formulas. This is an example in point where Alice and Bill feel that their students should be exposed to some mathematical rigor and reasoning, whereas Carl is of the opinion that the formulas are handed to students during an exam ready for use. These positions suggest that Alice and Bill aim in their

teaching at the long term benefits of their efforts, whereas Carl's effort focuses more on immediate goals, like instilling in students a feeling of achievement or on student interest which is essential for their success at the academic level. There was general agreement among case and sample teachers that when a formal proof was given, it unified basic ideas or in such instances the proof was an interesting example of mathematical reasoning.

Sample teachers used primarily the ratio interpretation and the unit reference circle interpretation for several reasons. It was emphasized in the Curriculum Guide, easy and enjoyable for students and it was expected to be on the MDE. The wrapping function interpretation and line interpretation were not familiar to teachers or considered too difficult for students and hence they were not included. Carl's use was identical to that, whereas Alice added the wrapping function interpretation to that and Bill used all four interpretations with his class. With respect to the reasons for using these interpretations, there were again differences between the case teachers and the case and sample teachers. Carl expected these interpretations to be on the MDE. Alice included them because they related closely to mathematics studies in prior grades and they were easy for students to understand. Bill cited the same reasons for inclusion as Alice, but added a third reason, the interpretations were included in the text.

The sample teacher responses on the Skill Emphasis Survey (SES) identified "skill in solving for missing parts of right triangles" and "skill in applying trigonometry to application problems" of primary importance. Alice and Bill presented a different view, suggesting that the skill in solving for missing parts of right triangles is already mastered by the students and therefore they did not emphasize this skill, whereas Carl agreed with the sample teachers. While there is general disagreement among the case teachers and between the case and sample teachers with regard to the relative importance of each skill, the case teachers emphasized substantially more skills in their classroom practices. The differences among teachers are similar to those cited in the above section dealing with trigonometry objectives.

Although there are again several differences among teachers, the

responses by the case teachers overlap substantially with the sample teacher responses on the Curriculum Material Selection Survey (CMSS) for most factors. There is agreement that "what students will need to know in order to do well on the mathematics diploma exam", "the guidelines given in the Mathematics 30 Curriculum Guide", "the students' ability", "the time available", "approaches that have worked well for me in the past" and "what students will need to know in order to succeed in their post-secondary education" are important factors that influence the selection of curriculum materials for use in the classroom and consequently the teacher's view of mathematics. The teacher responses suggest that factors in close proximity to teachers seem to have the greatest influence on their interpretation of curriculum materials. In contrast, factors like, "What society needs by way of informed citizens" or "What students require in order to understand the historical development of mathematics" received very low ratings in terms of their importance.

The time allocated for the trigonometry unit by sample teachers ranges from 25 to 35 hours with a mean of 29 hours. The findings are similar for case teachers, except for Bill who spent 40 hours in trigonometry. With respect to the time allocated for each of the twelve major topics or activities that comprise the trigonometry unit, the responses by the case and sample teachers show considerable variation. This suggests that teachers' views of mathematics vary significantly as is evident from the instructional emphasis given to the various topics. These variations among teachers are the result of conscious decisions they make and they are motivated by basic needs arising in their working environment. For example, Bill's choice of allocating nearly twice the recommended number of hours is probably related to the fact that trigonometry is his favored topic but also students are telling him that "...trigonometry is the most interesting and practical unit in Math 30 ". This suggests that student-related factors have a significant impact on the interpretation of curriculum materials. But, aside from that the case teachers expressed on numerous occasions that, while a specific curriculum is in place, they enjoy the flexibility that they have within it. Since the student-related factors may be significant for some teachers, they need the flexibility to respond to these student needs.

Teaching: The perceptions of teaching of the sample and case teachers are significantly shaped by their strongly held value relative to student understanding and their success on the MDE. As a result, the inclusion of topics and the depth and breadth of coverage of these topics is guided by the curriculum guide, the MDE and by what is useful for further study in mathematics. The use of a particular trigonometric interpretation is a case in point. The ratio and unit reference circle interpretations are included because they are in the curriculum guide, assumed to be on the MDE and useful for further study in mathematics. The inclusion of the wrapping function and line representation of trigonometric functions varied among teachers and depended on a number of factors such as the teacher's own background, difficulty level for students, ready access of material in text and it being a curriculum requirement. Although there is generally close adherence to the core curriculum, there are also instances where the sample and case teachers go well beyond and below the expectations set out in the curriculum guide. This is particularly evident from the responses to the TFQ which reveals that teachers do formally prove many of the trigonometric identities instead of deriving and applying them as stated in the curriculum guide. In other instances, teachers did not cover certain formulas when, in fact, it was a requirement in the curriculum. This, in turn, has an effect on the expectations of students which vary significantly among the teachers. Some require formal proofs of some identities and others expect students to apply the identities when given. The variations in expectations seem to be reflections of different goals for different students. As is indicated elsewhere in this study, the case teachers described their students as having wide-ranging abilities and one way of responding to this is by adjusting expectations or modifying content. In general, sample and case teachers value high student expectations and academic rigor as is reflected in the responses. Another value that is commonly held by the sample and case teachers is pedagogical efficiency. This is most evident in their selection of instructional strategies. Teacher demonstrations, class discussions, and drill and practice are clearly the prevailing teaching strategies. Instructional strategies that contain elements

of student explorations and investigations, such as problem solving in the context of real applications, use of computers, library research, math-lab experiments or group projects, are never or rarely used by teachers. Although these activities are by nature more enjoyable for many students, teachers view them as time consuming and impractical, given the amount of content to be covered. Teachers fear that the achievement goals, often set externally, would not be met if these strategies were used more frequently.

With reference to the findings on the RMS, the sample and case teachers tend to use the prescribed text, curriculum guide and teacher-prepared materials most frequently in their classrooms. This provides them with some assurances that the materials selected will be reasonably congruent with the curriculum objectives and ultimately with the MDE. The adherence to these resources also serves the purpose of being a protective device against those to whom the teachers must account. However, these resources, while at times confining to teachers, serve important purposes: for lesson preparation, for long-range planning and for student evaluation. With these elements being taken care of, teachers are then free to exercise flexibility in other areas of their teaching. The findings for the sample and case teachers suggest that the strongest influence on the selection of curriculum materials, and ultimately the teaching strategies, emanates from the MDE. That is likely because the MDE is seen as eroding their flexibility and independence in teaching and the treatment of the curriculum content. Other factors of influence on the curriculum material selection that are shared between the sample and case teachers are: the students' ability, approaches that have worked well in the past, what students will need to know in order to succeed in their post-secondary education and what students require in order to understand the structure of mathematics. These curriculum choices are based on the needs of the students and those of the teacher. While the sample and case teachers share a number of common values, beliefs and perceptions about teaching, the interpretations of the curriculum as reflected in their actual classroom practice seem to differ significantly. In examining the responses of the sample and case teachers to the TAQ, the point becomes obvious. For example, the case and sample

teachers closely adhere to the core curriculum, their curriculum material selections are strongly influenced by the MDE and the curriculum guide, and for the most part they are using similar resources and yet, there are wide ranges in total time allocated and even greater variations in the time spent on selected topics that make up the trigonometry unit. These findings seem to suggest that when teachers share common values, beliefs and perceptions about teaching, it will not necessarily translate into similar interpretations of the curriculum or similar classroom practice. This is clearly a reflection of teachers making conscious choices based on the particular needs within their working environment.

Students: The responses of case and sample teachers indicate that their perceptions related to students are the same.. They describe the general attitude of their students toward learning mathematics as well as their understanding of the material in the trigonometry unit as "good". The teachers' relationship with their students is characterized by a strong focus on the students' interests and abilities. This focus on student interest and ability is motivated by two important needs. The first need is that of the students who prefer interesting activities at their level of ability. The second need is that of the teacher who will derive positive feelings and feedback from having satisfied the students' needs.

School Setting: Many similarities exist between the case and sample teachers relative to the school setting. With regard to such factors as the number of different courses taught, the number of students that teachers come in contact with and the number of mathematics courses taught, the similarities between case and sample teachers are very close. Every sample teacher teaches at least four courses and one teacher teaches seven different courses, not all of which are mathematics courses. The number of students seen by sample teachers ranged from 50 to 150 students. Even Carl's particular case which involves teaching seven different courses and seeing 124 different students daily falls within the range reported by sample teachers.

The satisfaction level with the physical setting in which mathematics is taught is again slightly higher for the case teachers. Other school setting factors like the grade arrangements within the school, staffing levels and the number of mathematics teachers seem to be similar to those found among the three case teachers. While sample teachers indicate that the use of electronic devices (eg. calculators and computers) for solving problems in trigonometry is of primary importance in their classroom, the actual use of computers in their classrooms occurs at a level similar to that of the case teachers as is confirmed by their responses to the SES, TSS and RMS.

This contrast of the case teachers against the sample teachers reveals that the perceptions of mathematics, teaching, students and school setting, and the factors that influence the teachers' interpretation of curriculum materials for use in the classroom are generally very similar for both. While the purpose of this comparison was to put the three cases within the provincial context, it has actually made the evidence from the multiple cases more compelling by showing a high degree of convergence of the two sources of data.

C. Relationship Among Influencing Factors

The fourth major research question raised at the cross-case level is stated and answered in this section using the findings at the case level.

Question 4: What is the relationship between the various factors identified in this study that affect the teachers' interpretation of curriculum materials?

Most of the factors and their influence on interpretation of curriculum materials are outlined in question 3 in Chapter 5. The data suggest that the values and beliefs under the four curriculum commonplaces have a strong influence on curriculum interpretation. While a set of values and beliefs emerged for each of the commonplaces, the data suggest that their influence is not confined to each of the commonplaces. For example, teachers' perceptions of school mathematics have a profound influence on their perception of

teaching or the students. Conversely, the values held on teaching shape the teachers' view of school mathematics. Community expectations is a strong school setting factor which influences the teaching strategies significantly and consequently the mathematics content. Among the case teachers a shift occurred from a process orientation in teaching toward a product orientation and the mathematical content was selected for success on the MDE. The situational factors within a school setting, such as number of mathematics teachers, the timetable, the teaching assignment and the nature of the students had a strong influence on the other curriculum commonplaces and consequently on the teachers' interpretation of curriculum materials. Each of the values and beliefs under one curriculum commonplace is intricately linked with the values and beliefs of the other curriculum commonplaces. This has also been confirmed in other research studies (Crocker, 1979; Connelly & Elbaz, 1980; and Olson & Russell, 1984) which suggest that the factors within the teachers' functional paradigm are closely intertwined.

It is not surprising that the elements related to teacher factors have a strong impact on the interpretation of curriculum materials since experienced teachers or teachers with a stronger mathematical background have normally a more extensive reservoir of ideas and accumulated resources. The data suggest that training in mathematics (ie. number of mathematics and methods courses) and experience in teaching mathematics had an effect on the teacher's perceived level of adequacy to teach problem solving, teach mathematical applications and to implement the intended curriculum. Teachers with less background and experience tended to have a greater need to interact with other mathematics teachers. The teachers' academic background tended to have an influence on their perception of school mathematics.

VII. WHAT MAKES TEACHERS TICK?

This chapter is structured into four major sections. In the first section the research findings are reported and discussed with reference to the literature reviewed in Chapter II. The next section examines the nature of teachers' functional paradigm as characterized by their interpretation of the mathematics curriculum. Recommendations for further research follow which include suggestions that would contribute to a better understanding of the teachers' interpretation of curriculum materials for use in classrooms. The final section presents some concluding thoughts.

A. Research Findings In the Context of the Literature

This section contains a brief review of the research findings relative to the first four major research questions.

1. What interpretation do teachers give of the intended curriculum as represented by the trigonometry unit of the Mathematics 30 curriculum?

The study found a significant gap between the intended curriculum and observed classroom practice for the three case study teachers. This finding is consistent with other research. (Hall and Loucks, 1982; McNeil, 1986) In this study the gap refers primarily to the achievement of non-content-oriented goals in mathematics and the nature of the MDE, but it does not exclude specific learning objectives. The findings of this study suggest that when goals or objectives are not measured, teachers will operate their mathematics class just to cover the material on the test. For example, finding the area of regular polygons, using trigonometry, was not covered because there had not been a problem on this on the MDE. In fact teachers have the tendency to think of the goals of instruction only in terms of content. Non-content-related goals seem to be largely ignored by teachers. These findings are consistent with the research by Wilson (1971). The following is an example where this is evident. The goal

of developing an appreciation of the contribution of mathematics to the progress of civilization is one such non-content goal. None of the three case study teachers made any reference to the rich historical development of trigonometry as described in the review of the literature or placed the content within the historical context.

This lack of emphasis of non-content-related goals is closely linked with the nature and strong influence of the MDE. This strong influence is obvious in the direct relationship between the teachers' emphasis on a specific goal or objective and the degree to which it is measured by the MDE. In actual practice this means that teachers confine themselves almost exclusively to the coverage of the core content since the MDE is a reflection of that content only. On occasion, teachers extend the core content for the purpose of motivating students or providing for academically talented students. The data further suggest a linking problem between the intended curriculum and the interpreted curriculum as measured by the MDE. The focus of the MDE is exclusively on learning objectives that describe specific content. Teachers generally adhere closely to the core curriculum, as defined by specific learning objectives, with an emphasis on those concepts, skills and procedures that are assumed to be on the MDE. This accounts for the exclusion of the elective component and the use of that time for covering core content. While the elective component is a mandatory part of the curriculum, it is not included on the MDE and therefore not covered nor evaluated by the case teachers. Even problem solving and applications are for the most part confined to activities that improve the students' proficiency in paper and pencil skills. For example, teachers focus particularly on questions of the multiple-choice format and open-ended questions that require the students to show the steps in the solution process. However, the open-ended questions are, for the most part, similar to the multiple-choice questions except that the response alternatives have been removed. Both types of questions are included on the MDE and teachers practice these with a strong emphasis on the product rather than the process. The goal that problem solving be emphasized in the context of real applications, using microelectronic technology, is far from being achieved. The findings suggest that problem

solving and applications are still treated as a separate activity which engages students in applying specific techniques to classes of word problems. The intended use of microelectronic technology is essentially limited to calculators. Computer use occurs on rare occasions. An emphasis on problem solving, applications and the use of computers in the instructional process represents a major change for classroom teachers. (Crocker, 1983)

In commenting beyond the current study, it is evident that such a major change requires considerable attention to the implementation phase if any positive change in the schools is going to be brought about by the proposed program. (Leithwood, 1981) The literature suggests that conventional inservice activities are totally inadequate to inform teachers of the intentions of the new program. (Werner, 1980; Fullan, 1982)

2. What are the classroom practices of teachers and to what extent are these practices successful?

Based on the three cases studied, teachers follow a well established daily routine consisting of reviewing previous material, presenting new material and student practice. The most dominant teaching strategies used by teachers in this study include demonstrations, class discussions, drill and practice, and checking homework on the blackboard. Both of these findings are consistent with other research. (IAEEA, 1985; Rosier, 1980; Robitaille & Sherrill, 1977) Teaching strategies that are seen as inefficient in terms of time are never or rarely used.

While the choice of teaching strategies is closely linked with pedagogical efficiency, so is the use of curriculum resources. Teachers prefer having curriculum resources that reflect the objectives in the curriculum guide. Teachers feel that if the emphasis is to be on problem solving, applications and the use of microelectronic technology, then curriculum resources should be available to teachers that are congruent with this emphasis. Teachers seem to have some discomfort with the selection of curriculum materials from multiple sources. The findings suggest that this discomfort is related to their lack of time, their uncertainty about appropriateness of material and the curriculum

adaptation problem. These concerns are significant in the lives of teachers when they translate curriculum material into classroom practice, as has been confirmed by other researchers. (Connelly & Ben-Peretz, 1980; Leithwood & MacDonald, 1981)

While the teachers in this study are calling for well-defined curriculum guidelines with matching resources, they do wish to retain the flexibility to deviate from the curriculum when the students' needs and interests call for it. Their current classroom practices suggest that teachers do some adapting of the curriculum to meet student needs and they consciously select different interpretations of the same material. However, these adaptations and interpretations are limited to a level that provides students with the prerequisite skills for the diploma exam. While these modifications are primarily motivated by the students' abilities and interests, the teachers' own perception of the subject is a strong influence as well.

Although the three teachers share a number of common values and beliefs about school mathematics, teaching and students, their program emphases and time allocations vary significantly. The following examples illustrate the differences among the three teachers. Alice spends about one-half hour on developing the concept of a wrapping function, Bill deals with this concept for four hours and Carl does not cover the concept at all. Similarly with the concept of periodicity in conjunction with graphing of trigonometric functions, Alice allocates three hours, Bill seven hours and Carl two hours. However, with respect to reviewing and maintaining previously learned materials, Carl spends more than twice the amount of time relative to the total time allocated than Alice and Bill. The three teachers also differ significantly in total time allocated to the trigonometry unit. While Alice is slightly below (25 hours) the recommended time (26 hours), Carl exceeds it by four hours and Bill is well beyond it with forty hours.

Differences among the three teachers also exist with respect to student evaluation and the integration of computers into the instructional process. In connection with student evaluation, there are three major differences. These differences center around the frequency, purpose and the nature of evaluations.

Alice and Bill would use frequent quizzes to assess student understanding, whereas Carl would not and yet, both positions are reflections of pedagogical efficiency, a value that is strongly held by all three teachers. However, the difference is that Alice and Bill view the quiz as an efficient diagnostic tool that gives them insight into the students' understanding of the mathematical skills and procedures taught, whereas Carl sees it as requiring valuable instructional time. With respect to the purpose and nature of evaluations, Alice and Bill design quizzes for the purpose of assessing skill development and their tests are administered for the purpose of assessing the broader conceptual understanding and the connection of various mathematical concepts. Carl, in contrast, views the quiz and test more as another practice sheet as is evident from the skill level questions on these assessment tools.

The differences relating to the use of computers suggest that the integration of computers in the mathematics classroom is dependent upon the teachers' own competence and ability to use them. Teachers who lack the background and training with them seem to deliberately avoid using computers in class for fear of losing that sense of being "in charge". Alice, who has the background with them, uses them for graphing trigonometric functions. In fact, she expressed frustration that they were not available to her on a permanent basis. Bill and Carl, who have access to computers, deliberately avoid using them because they have no experience with them. While the use or non-use of computers is somewhat determined by the teachers' experience with them, their non-use is based on the belief that teachers are as effective teaching a mathematical concept without them.

The classroom practices of the case teachers revealed a number of strongly held values. The case teachers value pedagogical efficiency, academic rigor, student understanding and success on the MDE, motivating students, high and clear expectations of students, mathematics as process, feedback on student performance and professional competence. Each one of these values impacts the interpretation of curriculum materials for use in the classroom and is itself shaped by the classroom realities.

The questions, "How should mathematics be taught?" (van Hiele, 1957,

1980), "How can one ensure that the students' mathematical knowledge in a specific topic is properly built?" (Kieren, 1981) and "How can mathematical meaning be taught?" (Sigurdson, 1988; Sigurdson & Olson, 1989) have considerable significance in terms of the teachers' classroom practices. However, these questions are only significant to teachers in terms of their espoused theories or theoretical knowledge about practice, but when it comes to their actual classroom practices or theories-in-use, there is little evidence that these questions are addressed by teachers in a meaningful way. (Argyris & Schön, 1974) The reasons for these obvious differences between the teachers' talk about the curriculum and the actual interpretation of that curriculum in practice are part of the teachers' functional paradigm and they are further discussed in section B of this chapter.

Finding ways to develop insight in their students, allowing students to experience the various thought levels or exposing students to the five phases of learning (van Hiele, 1957) are not reflective of the teachers' classroom practices. This is a dilemma that is closely linked with the student assessment practices and it will remain such until they are properly aligned with the goals, objectives, content and instructional practices. (Wilson, 1971; NCTM, 1989)

The extent to which the case teachers succeeded implementing their interpretation has been examined using two measures of assessment: students' attitude toward mathematics and their achievement on the MDE. The overall results on the ATMT reveal that students rate the importance of mathematics higher than the degree to which they liked it. A linking of the ATMT results with the classroom observations suggests that students rate topics or classroom activities as important if it is seen or treated by the teacher as important. However, students may not necessarily like something that they have rated as important. The results further reveal that activities that are closely linked to the MDE received high "importance" ratings. This is again consistent with the observations because teachers generally emphasized content that they thought would be on the MDE. It was not uncommon for teachers to alert students to mathematical content that they anticipated to be on the MDE. While the overall student attitude toward mathematics was generally positive,

Mathematics in School was seen as memorizing trigonometric formulas and theorems and checking answers. This view of school mathematics by the students is congruent with the actual classroom practice, which consisted largely of drill and practice and checking answers to exercises. The section of the ATMT dealing with Mathematics as a Process is summed up by students as a set of rules to be followed. However, students feel that these rules help them to think logically. The results to Mathematics and the Students suggest that they feel good about solving problems by themselves, that they want to do well in trigonometry and that they have a good understanding of trigonometry. With respect to student achievement on the MDE, the data indicate that the achievement in trigonometry alone ranges from slightly to significantly below provincial average.

With respect to achievement on the total MDE, the results range from significantly below to slightly above the provincial average. While there are clearly links between the instructional practices of the case teachers and the results of their students on the ATMT and MDE, it is difficult to provide an explanation without considerable speculation. In addition, the absence of data on a number of relevant variables would render such an explanation inadmissible. However, the results do suggest that the success in interpreting the intended curriculum depends to a large measure on how closely the assessment methods and teacher interpretation align with the intended curriculum. Such alignment is very important, because "... little information is produced about students' mastery of curricular topics when the assessment methods and tasks do not reflect curricular goals, objectives, and content; the instructional emphasis of the mathematics program; or how the material is taught". (NCTM, 1989)

3. What are some of the factors which influence the teachers' interpretation of the trigonometry unit to students?

Two major areas of influence on the teachers' interpretation of curriculum materials are reported. One major area includes the values and beliefs associated with the four curriculum commonplaces and the other area is

associated with teacher factors. The values and beliefs associated with school mathematics and teaching surfaced as the most dominant influences, followed by influences related to students and the school setting. These findings are consistent with other research studies (Leithwood, Ross & Montgomery, 1982; Wahlstrom, Regan & Jones, 1982; MacDonald & Leithwood, 1982; Fullan, 1982; Connelly & Elbaz, 1982; Thompson, 1983; Sigurdson & Olson, 1989) in that they confirm that the strongest influences on teachers' interpretation of the curriculum emanate from within the classroom. Thompson (1983) found that mathematics teachers' conceptions; their views, beliefs, values and preferences about the subject matter and its teaching play an important role in affecting their effectiveness as the primary mediators between the subject and the learners.

The current study identifies seven factors related to mathematics and eight factors related to teaching that affect the teachers' interpretation of curriculum materials. Although this study reveals only three student related factors, they are however, strong influences on the teachers' classroom practice. The student interest and ability factors have been confirmed in studies by Fullan (1982); Wahlstrom, Regan & Jones (1982); Lantz (1984); and Sigurdson & Olson (1989).

In relation to teaching, the study by MacDonald and Leithwood (1982) suggests that teachers have an overriding need for achievement, defined in terms of assisting students to become more cognitively competent. The implications are that, given achievement as the core need and students' cognitive growth as the sole criterion of success, teachers will continue to direct their energies toward students' success on the MDE. Although the MDE is identified almost as a strong negative influence on teachers' curriculum interpretation, it does represent a useful source of confirmation about student progress and it does contribute to the fulfilment of the teachers' achievement needs. Inherent in the new curriculum is a call for a change in classroom practices, however, this change will not occur as long as the MDE is not aligned with the goals, objectives, mathematical content and instructional approaches of that curriculum. Indeed, the classroom practices will gravitate toward the demands of the MDE as is so strongly expressed by Bill. Three factors also

emerged in the area of school setting. The lack of equipment and resources, community expectations and situational factors (eg. number of mathematics teachers, teaching assignment and the nature of the students) are identified in the study. Some of these are also identified by Robitaille and Sherrill (1977).

Teacher factors are identified as the second major area of influence. Specific factors with the most influence on teachers include training in mathematics and experience in teaching mathematics which is also confirmed by other researchers. (Leithwood, Ross & Montgomery, 1982; Lantz, 1984)

4. What is the relationship between the various factors identified in this study that affect the teachers' interpretation of curriculum materials?

The findings of the current study suggest that the factors related to teaching have a strong impact on translating curriculum materials into classroom practice. Mathematics teachers consciously select instructional strategies that emphasize students' understanding and their success on the MDE, pedagogical efficiency, academic rigor, motivating students, high and clear expectations, mathematics as a process, student feedback and that ensure a display of their own professional competence. While these factors are common across the teachers, the activities that reveal them are often quite different. In contrast, classroom practices that do not comply with these values and beliefs are seldom or rarely chosen by teachers. For example, the use of library research, mathematics laboratory experiments or the discovery approach as a way of teaching mathematics would rarely be used by teachers. Not only are these approaches being seen by teachers as inefficient and lacking academic rigor, but they also give teachers a reduced sense of control over content as well as classroom management. All of these factors and those associated with school mathematics, students and school setting shape the use of curriculum materials in the classroom and ultimately the teachers' philosophy.

According to the data, the teacher factors, like mathematics background and mathematics teaching experience, have an influence on the teachers'

interpretation of curriculum materials and on their view of school mathematics. The data also suggest that an important component of the teaching situation is the number of mathematics teachers at school, the number of students, the teaching assignment and the time table.

The findings of this study also suggest that the factors associated with each of the two major areas of influence are not mutually exclusive, but rather, they affect each other in a complex way. The teachers' view of school mathematics is closely linked with their teaching practices and these, in turn, are related to the nature of the students and various situational factors.

As the teachers' interpretations unfold, curriculum developers and supervisory personnel have the tendency to view it as something to be changed rather than regarding it as an important resource to effect change. (Connelly & Ben-Peretz, 1980) Since the teachers' interpretations are invariably a reflection of the practical realities of the classroom, it becomes a useful tool to make changes in course content and curriculum materials.

B. Nature of Teachers' Functional Paradigm

The last major research question is central to the current study and it is: "What is the nature of teachers' functional paradigm as characterized by their interpretation of the Mathematics 30 curriculum?" This study identified major areas of influence and specific factors considered important by teachers when interpreting curriculum materials. In this section the focus is directed away from further describing "how" teachers function and interpret curriculum materials to "why" they function as they do and the bases for their decisions. However, the answers to the "why" questions are largely drawn from the answers to the "how" questions which involve a tracing of the factors that teachers consider important in their classroom practices.

The first part of this section deals with general features of the teachers' functional paradigm which include the elements that contribute to the functional paradigm of mathematics teachers and their influence on interpreting curriculum (Table 3) as well as the context of teachers' functional paradigm

(Figure 4). The second part of this section identifies a number of characteristics of the teachers' functional paradigm and describes each briefly.

General Features of Teachers' Functional Paradigm

The current study was conducted believing that the introduction of the new mathematics curriculum and the requirement of students to write the MDE presented a "strategic site" for the study of teachers' functional paradigm. The program structure and emphasis were significantly different from the previous curriculum, in that the core-elective format was introduced and the emphasis was to be placed on problem solving, applications, the use of microelectronic technology and providing for the needs of individual students in a flexible way. At the same time, the provincially administered MDE became a highly visible tool to measure student achievement and to set a provincial standard. Curriculum developers clearly introduced significant changes not only in mathematical content, but also in the way that this content was to be taught and evaluated.

It is in this context that the nature of the teachers' functional paradigm is being examined. Table 3 contains a summary of some of the beliefs and values of the three teachers and their corresponding use of curriculum materials in the classroom. The use of Schwab's four curriculum commonplaces is not to imply that these categories are distinct in the minds of teachers, but rather they facilitate the discussion. It is recognized that these commonplaces are closely interwoven which has been confirmed by other researchers as well. (Crocker, 1979; Connelly & Elbaz, 1980; Lantz, 1984)

The functional paradigm of teachers takes into consideration a number of influences. These are schematically shown in Figure 4 which suggests that these influences emanate from policy documents, curriculum resources, teacher factors and the teaching situation. Together, they act upon the teachers' values, beliefs and perceptions which ultimately shape the teachers' classroom practice. Given the existence of the teachers' values, beliefs and perceptions about the four curriculum commonplaces (Table 3) and their matrix of decision-

making (Figure 4), the question of the teachers' functional paradigm can now be answered by tracing the various factors that teachers consider important in their classroom practice.

In answering the question, "What is the nature of the teachers' functional paradigm as characterized by their interpretation of the Mathematics 30 curriculum?", the thinking is redirected toward explanatory principles rather than a description of classroom functions. (Crocker, 1983) Questions like, "Why do teachers think like they do?", "Why do classrooms function as they do?", "What information do teachers find valuable?", "How do teachers know whether what they are doing is correct?", "What kind of feedback do they rely on?", "How do teachers arrive at their philosophies?", "What knowledge paradigm do teachers work under?" and "How complicated is the teachers' matrix of information and knowledge?" are being examined when describing the teachers' functional paradigm. In summary, the question of "What is the teachers' functional paradigm?" is the question of "What makes teachers tick?".

Outstanding Characteristics of Teachers' Functional Paradigm

The teachers in the current study share a number of common goals, problems, techniques, exemplars and routines that constitute their functional paradigm. An examination of the questions raised above led to the identification of some characteristics that are part of the teachers' functional paradigm.

Skepticism: While the introduction of the new curriculum and the mandatory MDE represented a challenge to the teachers' prevailing paradigm, the evidence suggests that teachers continue to use their established practices to meet the new challenges. However, the evidence also suggests that the new policy documents and curriculum resources do have an impact on teachers. This impact, however, seems to be, at least initially, on the teachers' espoused theories only and not on their classroom practices. What is implied here is that teachers typically examine a new curriculum from the perspective of their

TABLE 3

**Elements Contributing to the Functional Paradigm of Mathematics Teachers
and their Influence on Interpretation of the Mathematics Curriculum**

Perception of Mathematics

1. Believe the program is limited to topics listed in core curriculum.
2. Believe program includes a personal list of topics.
3. Believe in mastering basic skills.
4. Value major theoretical concepts and principles.
5. Value mathematics as a unified whole.
6. Value mathematics as a problem-solving tool.
7. Value the nature of mathematics.

Use of Curriculum Materials

- Limit selection of materials to topics in core curriculum.
- Omit elective topics as part of the course.
- Extend core topics beyond core curriculum according to personal taste.
- Review verbally and through worksheets and quizzes with focus on basic skills.
- Extend core topics to increase depth of coverage of theoretical concepts and principles.
- Review content and concepts to show the logical connection within the overall structure.
- Use problem-solving techniques to solve mathematical applications.
- Use mathematical content to illustrate the processes and to teach inductive and deductive reasoning, the nature of proof, power of mathematics and philosophy of mathematics.

Perception of Teaching

1. Value student understanding and success on the MDE.
2. Value pedagogical efficiency.
3. Value academic rigor.
4. Value motivating students.

Use of Curriculum Materials

- Use of previously written MDE to aid understanding and success on the MDE.
- Limit content to core curriculum.
- Design materials and tests which can be marked and returned to students quickly.
- Omit elective topics from the program.
- Use efficient teaching strategies to present material (eg. lecture).
- Use supplementary resources to challenge students.
- Supplement with more detailed information and notes.
- Increase pacing of lesson and activities.
- Vary class activities for students.
- Present interesting challenges for students.

5. Value high and clear expectations of students.
6. Value mathematics as a process.
7. Value feedback on student performance.
8. Value professional competence.

- Require homework to be done.
- Select topics and activities that challenge students.
- Keep program content the same for all students.
- Emphasize exploratory activities.
- Stress problem-solving process.
- Use evaluative activities (eg. verbal or written).
- Limit treatment of topic or discussion to teacher competence/comfort level.
- Select activities that worked well in the past.
- Select activities based on personal interest and background.

Perception of Students

1. Focus on students' interest.
2. Focus on students' ability.
3. Focus on students' success/in future careers.

Use of Curriculum Material

- Select topics and activities that students enjoy (eg. computers).
- Modify materials and approaches to suit students' ability level.
- Select curriculum materials to ensure proper prerequisite skills.
- Stress the importance of being successful on the MDE.

Perception on School Setting

1. Focus on school facilities.
2. Believe expectations of community are an important consideration.
3. Believe situational factors are an important consideration.

Use of Curriculum Material

- Restrict student activities because equipment and proper resources are unavailable (eg. computers, mathematical application and problem solving resources).
- Use materials or teaching strategies that will aid student achievement.
- Adopt a product-oriented approach with regard to curriculum materials and teaching strategies.
- Use limited/expanded resource preparation because of school resources, number of mathematics teachers, school timetable, teaching assignment (ie. number of students and subjects) and nature of students.

Context of Teachers' Functional Paradigm

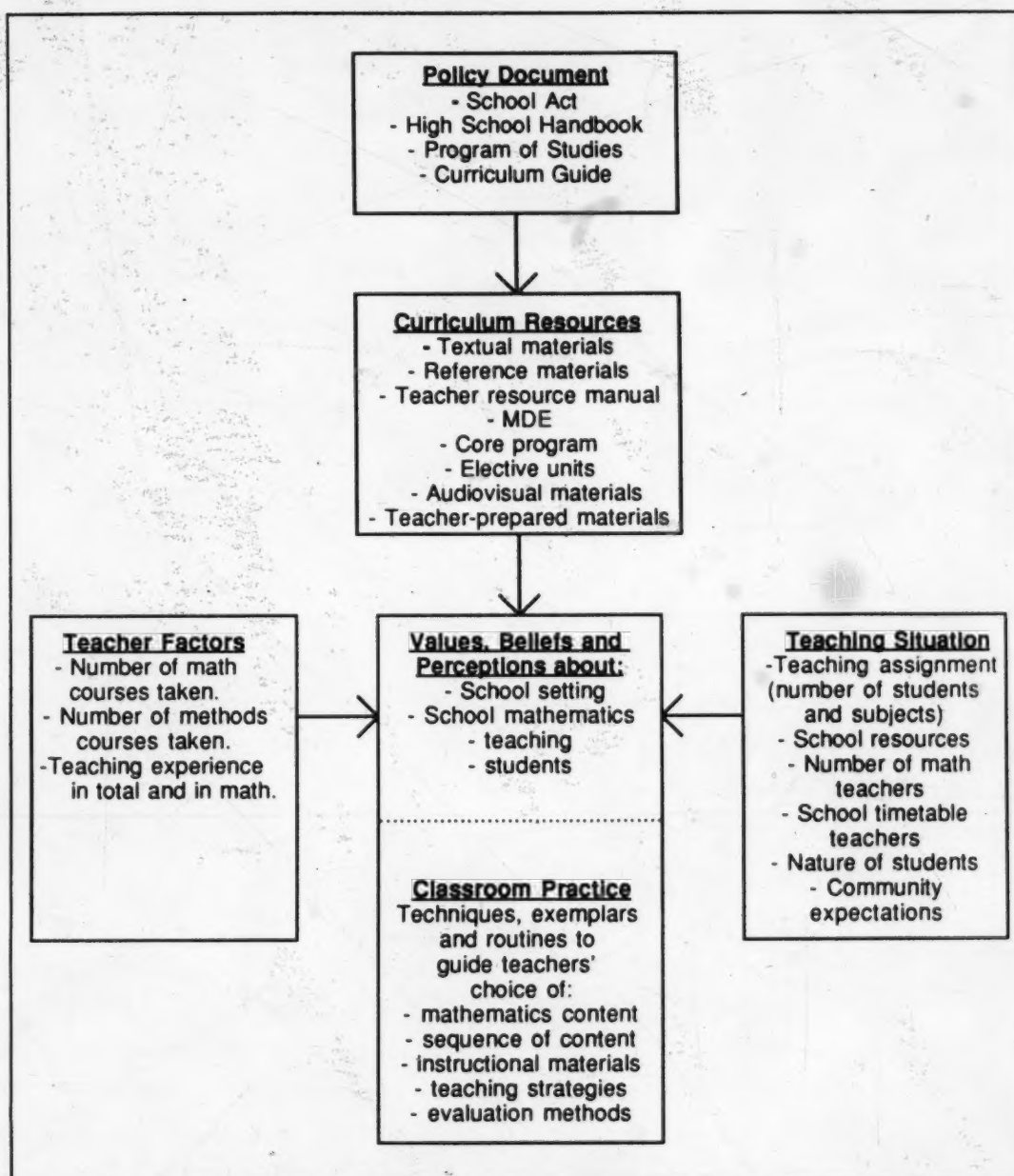


FIGURE 4

Context of Teachers' Functional Paradigm and the Effects on Classroom Practice

prevailing paradigm. They continue using their established classroom practices and they even mould the new demands into their existing functional paradigm, while simultaneously adopting the language of the new curriculum. It is quite common for teachers to say: "I know the new curriculum wants me to do this or emphasize that, but I prefer doing it the old way." For teachers to say this means that they do not expect the new curriculum to be different or that it demands new approaches. They tend to approach curriculum change with an attitude of skepticism. This attitude, for some, has been shaped by a history of curriculum changes which were often inconsequential in terms of teachers' classroom practices. As a result, teachers' feelings range from rejection of the new curriculum, or parts of it, to disbelief that the new curriculum is different or even correct. However, they remain skeptical with respect to the effectiveness of the new curriculum and the need to translate their rhetoric into practical action plans. Teachers are reluctant to adopt new practices, thus surrendering their security and comfort, when the old practices are still adequate to solve these new challenges.

As a general principle, teachers bring at first a new innovative curriculum in line with their own beliefs and values instead of the other way around. There are a number of specific examples that illustrate this point and the statement that teachers continue to use their established practices to meet the new challenges. The curriculum developers intended that problem solving be emphasized in the context of real life applications and that it be integrated into the overall program rather than be dealt with as an independent unit. However, problem solving continues to be dealt with as a separate activity with an emphasis on reaching "the right answer" instead of developing the skills and processes that can be internalized and applied to more than one type of problem. Teachers see no need to change because the MDE continues to emphasize "the right answer" instead of the process. For example, Bill, who favors a process orientation, feels that he penalizes his students by doing what the curriculum states he should do and consequently he has shifted increasingly toward a product orientation. The absence of an agreed-upon definition of problem solving and our lack of

understanding the nature of problem solving have not helped in diminishing this skepticism.

The use of computers and calculators is another case in point. Both are strongly recommended to be used yet, Bill and Carl, who are not using computers at all, are not experiencing any disadvantages on the MDE for their students. While Bill and Carl indicated that their non-use of computers is related to their lack of background and experience with them, in actual fact their non-use does not prevent Bill and Carl from meeting the challenges or solving the problems of the new curriculum. What is suggested is that the decision to use or not use computers is made on the basis of whether or not it is necessary to use them and not on the basis of their background.

The espoused theories of the curriculum developers as expressed in the curriculum guide are not only incongruent with the evaluation practices and tools used, but also with the classroom practices of the practitioners. These issues are, of course, closely connected.

The MDE is intended to reflect the objectives of the new curriculum, but it is achieved only in terms of the content objectives. Other objectives, like developing problem-solving skills in the context of real-life applications, developing skills in the use of computers while solving problems, developing positive attitudes towards mathematics or developing an appreciation for the rich cultural heritage and dimension of mathematics, are not measured or continue to be measured in the same way. Teachers recognize this incongruence between the intended curriculum and the assessment tool that measures the achievement of the intended curriculum, thus becoming even more skeptical. Their skepticism is also evident with respect to specific curriculum objectives, questioning whether they ought to be included and whether they are achievable. The objective to develop an appreciation of the contribution of mathematics to the progress of civilization and to emphasize problem solving in the context of real-life applications using computers are two examples of unrealistic objectives in the opinion of teachers.

Assessment-Oriented Teaching: When there is incongruence between the intended curriculum and the assessment tool that measures student achievement of the intended curriculum, the operating principle of the teachers becomes one of meeting the requirements of the assessment tool rather than the curriculum.

Hence for the practitioners it is a conscious choice to deviate from the curriculum and to focus on what is required on the MDE. The Math 30 Curriculum Guide, which is the intended curriculum, does guide teachers in terms of the breadth, depth and nature of the mathematical content that is offered to students. However, a much stronger influence than the curriculum guide is the MDE and what students will need to know in order to do well on it. This suggests that teachers will de-emphasize, even omit, content that is in the curriculum guide, but not on the MDE. The reason being that the MDE has a high public profile across the province and the results from it are often being subjected to various interpretations. The most threatening interpretation to teachers is the equating of the MDE results with their effectiveness as mathematics teachers. This clearly suggests that it is not the curriculum guide that dictates what teachers do in the classroom, but rather the nature of the MDE and how success or achievement is being measured. Copies of previously written MDE are extensively used by teachers, which serve them as information base for decision making. However, while the teachers' own need to achieve through the performance of the students on the MDE is very important, their reputation as a mathematics teacher and their genuine concern not to place their students at a disadvantage, are the primary reasons for such a strong focus on the MDE. This strong influence of the MDE accounts at least, in part, for the existence of two beliefs about mathematics - the belief that the program consists of the core curriculum and the belief in mastering basic skills.

Teacher Rhetoric: The functional paradigm that governs the teachers' view of mathematics is also closely linked with their view of teaching. While the study revealed that teachers hold a number of values about teaching, the underlying reasons for holding these values must invariably be found outside the teachers'

rhetoric. Frequent teacher inservice and other professional development activities have provided opportunities for teachers to align their espoused theories with those expressed in the curriculum. The number of mathematics and methods courses as well as the teaching experience in mathematics provide teachers with additional opportunities to refine their espoused theories about mathematics and teaching. However, while the "teacher talk" is the same as that expressed in the curriculum documents, the actual teacher practices often differ significantly. When there is a major curriculum change, mathematics teachers are anxious to bring their rhetoric in line with the new expectations, thus giving those to whom they have to account the impression that adoption has occurred. However, teachers' established practices continue in isolation from their rhetoric. The value that teachers place on student understanding and success on the MDE is an example of that. There is much teacher rhetoric surrounding this value, but little evidence that teachers consciously focus on teaching mathematical understanding in a general way. Mathematical understanding is clearly confined to providing students with the skills to succeed on the MDE. For example, teachers focus on core content only, review previously written MDE, do not offer elective topics and they omit topics not assumed to be on the MDE. The governing principle used by teachers is that content which is not evaluated is not important and does not contribute to the teachers' achievement needs nor to the students' success on the MDE. This is not to say that teachers view specific content as unimportant per se, but rather, their judgment is merely with reference to the requirements of the MDE. Closely related to this principle is the tendency for teachers to think of the curriculum goals only in terms of content because the MDE reflects content objectives only.

Other examples of where the teachers' rhetoric is not congruent with their practices is in the areas of problem solving, meeting individual needs and student evaluation. Problem solving continues to be a product-oriented activity instead of teaching students to formulate problems; apply various strategies to solve, verify and interpret the results; and generalize solutions. Although teachers talk about meeting individual needs of students, the observations suggest that, in general, all students are subjected to the same content. The

same observations can be made with respect to student evaluation. Mathematics teachers talk about student assessment for the purpose of diagnosing areas of difficulty for individual students or instructional planning, however, in reality assessment is for the sole purpose of assigning grades.

Self-Protection: The evidence in the current study suggests that teachers never or rarely use instructional strategies that involve small group activities, individual projects, library research, math-lab experiments, discovery approaches and other strategies that actively involve students and foster independence. Instead, teachers use primarily teacher demonstrations, drill and practice, and checking assignment questions. While teachers indicated that pedagogical efficiency is the primary reason for these choices, the findings suggest that this is only a small part. A more compelling reason is that all mathematics teachers have a strong affinity toward structure and their direct involvement in the teaching-learning process. The use of math-lab experiments, for example, requires that students assume some responsibility for their own learning and understanding of the concepts. However, teachers are unsure of and they have little trust in students being capable of extracting the right kind of knowledge from these activities. The evidence suggests that teachers feel that only when they have directed every step of the teaching-learning process or disseminated all knowledge to be acquired, that the content has been taught. This behavior is closely linked to the accountability issue and the need for teachers to protect themselves from claims that certain concepts have not been taught.

Other reasons related to the use of demonstrations and drill and practice as dominant instructional strategies are based on management, discipline and control, concern for efficiency in covering content and the inability of many students to work independently. Again, there is considerable variance between the practices of the teachers and those envisioned by the curriculum developers. While the goal of the curriculum is the development of the students' ability to become independent problem solvers, the prevailing classroom practices continue to foster student dependence on teacher-guided instruction.

The use of well-established classroom routines like checking assignments, reviewing, teaching new concepts and student practice have become important security blankets for the students and teachers. For the students they provide a high degree of predictability and thus comfort in knowing the rules governing classroom practices. For the teachers, the use of these routines reinforces the notion that the students are in a subordinate role and that the teachers are in charge. The nature and substance of each phase of these classroom routines is determined largely by the teachers and students have not only come to accept it, but to expect it and to like it.

This is further illustrated by the fact that mathematics teachers favor a high degree of structure and prescription. They want a clearly defined curriculum that sets out learner expectations and which is accompanied by preferably a single resource that matches that curriculum. The teachers in the current study expressed high levels of dissatisfaction with the text because of its inadequate coverage of the curriculum expectations. The use of multiple resources is not the teachers' preferred choice. The reasons that they cited included a concern for time and the inconsistencies in language between different resources. The findings suggest that their concern with multiple resources is also strongly related to their uneasiness and uncertainty about making their own choices.

Teachers prefer highly predictable, well planned and structured situations which reduce the number of choices they have to make as long as they have the flexibility to deviate from these structured situations when circumstances call for it. The reason being, that the more choices are left to be made by teachers, the greater the possibility of being criticized or questioned by the community or supervisors. Following an established structure or a prescribed plan provides teachers with the self-protection.

Tension: The notion of tension as a characteristic of teachers' functional paradigm is evident in several areas. While mathematics teachers expressed a strong desire for structure and prescription, they indicated at the same time that they wanted the freedom and flexibility to choose among the suggested topics

and activities to meet the needs of a particular class or individuals. The call for structure and prescription seems to be based on the teachers' desire to save time in preparing materials, reduce their responsibility relative to sequencing topics and have reliable assessment tools available. Their call for flexibility seems to be based on teachers' need to deviate from the curriculum when the situation calls for it. Teachers clearly want to have the flexibility to respond to questions that are beyond the topic or the curriculum, they want the flexibility to utilize teachable moments and they want the flexibility to ascribe meaning to curriculum materials that is based on their own perception of mathematics.

Tension is also evident between the teachers' rhetoric, which is the ideal, and their actual classroom practices. According to their rhetoric, teachers are concerned about students' understanding of mathematics, however, in actual practice this concern for understanding is confined to the concepts and skills tested on the MDE. Teachers' rhetoric about problem solving is particularly prevalent with little evidence that it is incorporated in their actual classroom practices.

The tension is also evident with respect to the question of whether teachers teach for the MDE or in preparation for university entrance. Teachers feel that there is a difference in that the former favors a product-oriented approach while the latter favors a process-orientation.

Teachers also experience considerable tension with respect to the depth of treatment of the content. Given that there is a wide range of abilities among the students, teachers tend to gravitate toward the "middle of the road" approach. This approach has often negative side effects that the teachers have to deal with. Those below the "middle" are frustrated because they still experience difficulties and those above are frustrated because it is not challenging enough.

Responsibility: The notion of responsibility is a very strong characteristic of the teachers' functional paradigm. There is no doubt that teachers assume responsibility for the teaching and learning of mathematics in a general way, however, they are reluctant to assume responsibility for each individual

student's achievement in mathematics in a very personal way. This would clearly be unbearable for teachers and hence they use practices that shift the responsibility to the clients whom they serve.

These classroom practices include a strong emphasis on teacher-directed learning, a high degree of structure and prescription and the selection of learning resources that match well with the curriculum. They also serve as a protective device against possible interference from the community or supervisors and as a "track" to which teacher can readily return following a brief excursion into a topic or activity of their own choice. However, the evidence from the current study also suggests that by using these practices, teachers are able to shift most of the responsibility for learning failures to the students and their families. Teachers can account with a clear conscience that they have directed the learning, they have adhered to the curriculum and in most cases they have assisted individual students by assigning additional exercises or by going over the same material again. The findings suggest that part of the teachers' functional paradigm is being moralistic, meaning that the lack of student success in the mathematics classroom is always traced to factors other than the teacher. The most commonly cited factors are the students' lack of application, students' interests, abilities and knowledge and a variety of school setting factors, like teaching assignment, school resources, school timetable, number of mathematics teachers, community attitude and lack of inservice. The strength of the teachers' moralistic disposition is directly related to the degree to which they are held responsible and accountable for the educational growth of students under their care.

Maintaining the System: While it appears that much of what mathematics teachers do in the classroom is influenced by students or student-related issues, like the MDE, the curriculum guide, the students' abilities, interests and what they needed to know in order to succeed in their post-secondary education, this characteristic suggests that mathematics teachers invariably engage in practices that are supportive of maintaining the system as opposed to serving the individual student. That means that mathematics teachers teach objectives

because the curriculum includes these or the school requires them instead of organizing instruction around the needs of the students.

The functional paradigm of mathematics teachers does not include modification of the curriculum to meet individual needs in a significant way. As a general rule mathematics teachers tend to assist those who experience difficulties by repeating the same instruction or assigning more exercises, hoping that students will eventually come to understand. There are at least two reasons for this approach. One reason is that the sequential nature of mathematics demands that a specific concept be understood before teachers can proceed to the next concept. The option of simply ignoring a difficult concept or substituting it with other simpler concepts is not available to mathematics teachers. As a result they keep on repeating the same instructions until the student understands or admits failure. The second reason for the lack of modifying the curriculum to meet individual needs is directly related to the teachers' unwillingness to take risks. Modifying the intended curriculum for an individual or a small group requires that teachers make professional judgments about the students and the mathematical content. Mathematics teachers are not prepared to take this risk for various reasons. An obvious reason is their inability to diagnose a mathematical deficiency and then prescribe appropriate remedial activities.

Another reason is their inability to deal with different mathematical streams within the same classroom. Teachers are talking about being guided by the students' abilities and their interests as it relates to planning for instruction, instructional strategies and student evaluation, however, the way teachers plan for and talk about these areas is significantly different from these plans in actual practice. Actual elements of these plans were only observed on a few occasions. For example, the plans made specific provisions for different abilities, however, in actual practice all students were subjected to the same "middle of the road" approach. Lack of understanding by a few students was invariably dealt with by repeating the same instructions, assigning additional practice exercises or, if the content was important, more time was spent on it. Consideration of students' abilities and interests is definitely part of the

teachers' functional paradigm, but there is no explicit provision of a differentiated curriculum in actual practice. All students are subjected to the same content and evaluation methods. The teachers in the current study suggested that content modification with a lowering of expectations was not really an option for them to meet needs of individual students, because in the end they all had to write the same MDE.

Placing the needs of the system over that of the individual students is further illustrated in the following example. Teachers indicated that their classroom practices are strongly influenced by what students will need to know in order for them to succeed in their post-secondary education. As a result mathematics teachers are constantly making reference to what students will need to know at universities, colleges or technical institutions. This reference to post-secondary institutions is frequently used as a justification for extending a particular core topic or including one of their own "pet topics". For example, the emphasis on formal proof is a case in point. While the curriculum guide does not require formal proof, teachers justify its inclusion by saying that it is expected of them at the various post-secondary institutions. The frequent reference by teachers to expectations of post-secondary institutions or those of the MDE is intended to serve as a motivational device that keeps students involved and on task and it is part of their functional paradigm. However, in actual fact it is more of a coercive strategy than it is motivational. Teachers use it because reasoning and other logical arguments are not always successful in convincing students to remain interested and involved. However, it does demonstrate that the teachers' support is toward maintaining the system in preference to supporting the individual student.

Uncertainty/Uneasiness: This element of the teachers' functional paradigm characterizes their world. There is uncertainty with respect to their interpretation of the curriculum and, connected to that, their selection of content for the purpose of instruction. Given that there are expectations that teachers make provisions for individual needs of students, their uncertainty and uneasiness are even further increased because this expectation demands the ability to

diagnose student difficulty, prescribe appropriate content and select effective teaching strategies. There is uncertainty and often uneasiness with respect to the degree of support from the community or supervisors relative to the decisions that teachers are making. Teachers experience considerable uncertainty and uneasiness with respect to the curriculum objectives and in some cases with the value of these objectives. For example, teachers are uncertain about problem solving in the curriculum because they do not quite understand what it means, how to teach it or how to evaluate it.

Given that teachers experience a high degree of uncertainty and uneasiness, it is therefore not surprising to see teachers extremely dismayed over the poor quality of a text or the poor match of resources with the curriculum. A good text, for example, reduces the uncertainty for teachers and minimizes the number of decisions. In that respect, the MDE has provided teachers with a great deal of guidance and certainty in curriculum decision-making. While they may still be uncertain or uneasy about their long-range goals, the MDE has provided clear direction as to what the immediate goals ought to be for teachers.

If it was the intent of the curriculum developers to bring about a change in the teachers' functional paradigm, then the new curriculum, along with the mandatory MDE, has not achieved this objective. It is obvious from the findings that the prevailing functional paradigm is still adequate to solve the problems posed by the new curriculum.

C. Recommendations For Further Research

This study is an attempt to gain a better understanding of how the intended curriculum is interpreted and why it is interpreted in particular ways. Therefore, the link between the intended curriculum and the interpreted curriculum provides the major context for the following recommendations.

1. Studies should be undertaken to examine the influence of mandatory external examinations (eg. MDE) on teachers' interpretation of curriculum materials and on students' learning practices.

2. Longitudinal studies should be undertaken to determine if mandatory external examinations limit or enhance the mathematical knowledge and scope of students over time.
3. Studies should be undertaken to investigate the influences of multi-grade or multi-subject assignments of teachers in shaping their functional paradigm.
4. Studies should be undertaken to determine whether any of the findings of the current study are generalizable to a larger population of mathematics teachers or to mathematics in general since this study confined itself to trigonometry.
5. Studies should be undertaken to investigate the role of training and experience in shaping teachers' functional paradigm.
6. Studies should be undertaken to examine the relationship between teaching strategies and such things as course content, nature of the students and teaching situation.

D. Concluding Thoughts

The findings in this study suggest that the interpretation of the mathematics curriculum is a function of the teachers' functional paradigm. The beliefs and values inherent in the functional paradigm are subject to strong external influences and they are ultimately affecting every aspect of teachers' classroom practices. The mandatory province-wide mathematics exam is one of many such strong influences on the teachers' classroom practices. The introduction of this exam, while intended to standardize the competency test for students, may actually create more problems than it solves. The evidence in the current study suggests that there is a significant shift toward teaching for such an exam.

These case studies offer a window into the effects of various factors on the teachers' functional paradigm, showing that the classroom practices are not kept in balance with all aspects of the intended curriculum. The deviations from the intended curriculum and the examples of questionable classroom practices seem to be the result of a vicious cycle which consists of reducing course content to its most manageable and measurable fragments, splitting the learning process into means and ends, and emphasizing the province-wide exam as the most important event for students and teachers. Evidence from the case studies suggests that a curriculum model which specifies certain curricular content and then determines the degree of student "learning" of that content ignores the fact that teachers will place the emphasis exclusively on inputs and outputs and not on the process in between the two. The program rationale, emphasis and goals of the intended curriculum reflect the rich cultural heritage and dimensions agreed upon by curriculum developers, but in actual classroom practice the translation of this cultural heritage occurs in forms that can be readily consumed by students for the purpose of succeeding on the diploma exam.

There is little hope for a paradigm shift among mathematics teachers as long as the assessment methods and uses are not aligned with the intended curriculum. A continued non-alignment of assessment methods with the intended curriculum will ultimately result in tightly defined content as reflected by the diploma exam and in the increased use of more efficient presentation strategies in the hope to improve achievements. Such a single focus - student achievement - ignores the interrelationship of the instructional strategies and instructional content. The current study suggests that the instructional strategies and the content selected affect the students' attitude and their knowledge of the content itself. The result is that students acquiesce to the pattern of classroom knowledge thereby participating in their own de-skilling of mathematical knowledge. What is needed is a closer examination of how content is processed, rather than focusing on the outcomes of that process as measured by the diploma exam. Only then can one begin to see the factors at work in content selection, in validating ways of presenting or encountering information

and those factors outside the classroom giving legitimacy to these processes.

In order for schools to be places where students have access to mathematical knowledge, they will have to be places where students' engagement in learning is merited by the mathematical content and the instructional strategies. However, for such mathematical content and instructional strategies to emerge, there need to be educational environments in which teachers teach against external assessment structures that are completely aligned with the intended curriculum.

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APPENDIX A

CLASSROOM OBSERVATION SHEET

Teacher's name: _____ School: _____

Day: _____ Date: _____ Time: _____
Subject: _____

Section 1: Physical Environment

1. Main Features (size, layout, etc.):

2. Windows:

3. Blackboards:

4. Tackboards:

5. A V Screen:

6. A V Equipment:

7. Mathematical Equipment:

8. Shelves:

9. Mathematical Displays:

10. Posters:

11. Desks:

12. Mathematical Resources:

13. Other:

Section 2: Teaching Strategies

1. Teacher Activity:
2. Student Activity:
3. Use of Texts, Worksheets, etc. :
4. Comments on Mathematics:
5. Comments on Students:
6. Comments on Teaching:
7. General Comments:

Section 3: Problem Solving Scale**Directions:**

1. Circle the number, indicating the rating for each of the four indicators, based on the defined observable behaviors.
2. Add the circled numbers to determine the score.

INDICATORS**Understanding the Problem**

- Asks the questions to clarify the problem
- States relevant facts in the problem
- Perceives implied relationships

Observed to a high degree 5.....4.....3.....2.....1 Not exhibited

Devising a Plan

- Summarizes data by making a table, graph, or diagram
- Develops approaches (eg. looks for patterns, works backwards, makes predictions and verifies, decomposes problem into parts)
- Recalls related problems previously solved
- Estimates solution

Observed to a high degree 5.....4.....3.....2.....1 Not exhibited

Carrying Out the Plan

- Uses a table or diagram arrive at solution
- Applies a formula
- Performs computation required for solution
- Decides where to begin
- Switches strategy when it is no longer applicable

Observed to a high degree 5.....4.....3.....2.....1 Not exhibited

Looking Back

- Describes strategy used in solving the problem
- Verifies that solution satisfies conditions of the problem
- Looks for alternative ways to solve the problem
- Creates applications or related story problems

Observed to a high degree 5.....4.....3.....2.....1 Not exhibited

SCORE: _____

APPENDIX B

CLASSROOM PRACTICES
of
TRIGONOMETRY TEACHERS
SURVEY
(CPTT)

Please write all your responses in the survey booklet. Do not write your name on the booklet. Use the enclosed envelope to return the completed survey to:

Klaus Puhlmann
Box 6482
Edson, Alberta
T7E 1T9

If you have any questions regarding the survey please phone Klaus Puhlmann at 795-2568

THANK YOU

This survey is part of a study that tries to identify the functional paradigm of Mathematics 30 teachers in interpreting the Math 30 curriculum into classroom practice. Particular emphasis is placed on the **Trigonometry Unit** of the Math 30 curriculum as stated in the Program of Studies for Mathematics 30 (yellow background statements in the 1983 Curriculum Guide).

Part 1: Background Information

This part seeks information about you and the school in which you teach. It will assist in interpreting your opinions.

For each question, circle the letter of the appropriate response.

(Note: Circle one letter only).

1. What is your sex?
 - a. Male
 - b. Female
2. What is your age?
 - a. under 26
 - b. 26 - 35
 - c. 36 - 45
 - d. 46 - 55
 - e. over 55
3. How many years of teaching experience (including the current year) do you have?
 - a. 1 year
 - b. 2 - 5 years
 - c. 6 - 9 years
 - d. 10 - 13 years
 - e. 14 years or over
4. How many years of experience (including the current year) do you have teaching high school mathematics?
 - a. 1 year
 - b. 2 - 5 years
 - c. 6 - 9 years
 - d. 10 - 13 years
 - e. 14 years or more

5. How many different courses are you teaching at present? (Consider Math 10, 20, 30 and Physics 10 as four separate courses.)
- 1
 - 2
 - 3
 - 4
 - 5 or more (please specify how many) _____
6. In the current school term, how many different students are you teaching?
- less than 50
 - 50 - 99
 - 100 - 149
 - 150 - 199
 - 200 or more
7. In the current school term, what percentage of your teaching load is comprised of teaching mathematics?
- less than 20 %
 - 20 % - 39 %
 - 40 % - 59 %
 - 60 % - 79 %
 - 80 % - 100 %
8. What is your area of academic specialization?
- Mathematics
 - Physical Science
 - General Science
 - Biological Science
 - Physical Education
 - Business Education
 - Other (please specify the area) _____
9. How many courses in mathematics (include Statistics and Computing Science) did you complete during your post secondary education? (Indicate in terms of full-year courses. A one-term or semester course would be considered half a course.)
- 1 or less
 - 1.5 - 3
 - 3.5 - 5
 - 5.5 - 7
 - 7.5 or more

10. How many courses in Curriculum and Instruction (methods) in mathematics did you complete during your post-secondary education? (Indicate in terms of full-year courses. A one-term or semester course would be considered half a course.)
- 1 or less
 - 1.5 - 3
 - 3.5 - 5
 - 5.5 - 7
 - 7.5 or more
11. How do you rate the adequacy of your background for teaching the elective units of the Math 30 curriculum?
- Very Good
 - Good
 - Satisfactory
 - Unsatisfactory
 - Very unsatisfactory
12. How do you rate the adequacy of your background for teaching mathematical problem solving (as defined in the Math 30 curriculum guide)?
- Very Good
 - Good
 - Satisfactory
 - Unsatisfactory
 - Very unsatisfactory
13. How do you rate the adequacy of your background for teaching mathematical applications (as outlined in the Math 30 curriculum guide)?
- Very Good
 - Good
 - Satisfactory
 - Unsatisfactory
 - Very unsatisfactory
14. How do you rate the adequacy of your background for teaching statistics using the experimental approach (see Program Emphasis section of the Math 30 curriculum guide)?
- Very Good
 - Good
 - Satisfactory
 - Unsatisfactory
 - Very unsatisfactory

15. How do you rate the adequacy of your background for implementing the overall philosophy, goals and objectives of the Math 30 curriculum?
- a. Very Good
 - b. Good
 - c. Satisfactory
 - d. Unsatisfactory
 - e. Very unsatisfactory
16. How would you rate the physical setting in your school in which Math 30 is taught?
- a. Very Good
 - b. Good
 - c. Satisfactory
 - d. Unsatisfactory
 - e. Very unsatisfactory
17. Generally, how would you describe the attitude of your Math 30 students toward learning Math 30?
- a. Very Good
 - b. Good
 - c. Satisfactory
 - d. Unsatisfactory
 - e. Very unsatisfactory
18. How would you describe the level of understanding of your students in trigonometry?
- a. Very Good understanding of the material
 - b. Good understanding of the material
 - c. Satisfactory understanding of the material
 - d. Unsatisfactory understanding of the material
 - e. Very unsatisfactory understanding of the material
19. What grades does your school include?
- a. K or 1 -12
 - b. 7 - 12
 - c. 10 - 12
 - d. Other (please specify) _____

20. What is the approximate number of full-time teachers at all levels in your school?

- a. 1 - 19
- b. 20 - 39
- c. 40 - 59
- d. 60 - 79
- e. 80 or more

21. How many mathematics teachers are there at all levels in your school?

- a. 1 - 2
- b. 3 - 4
- c. 5 - 6
- d. 7 - 8
- e. 9 or more

22. What is the total enrolment of your school?

- a. less than 500 students
- b. 500 - 749 students
- c. 750 - 999 students
- d. 1 000 - 1 499 students
- e. 1 500 or more students

23. What is the population of the community (ie. city, town or village) where your school is located?

- a. less than 1 000
- b. 1 000 - 4 999
- c. 5 000 - 29 999
- d. 30 000 - 99 999
- e. 100 000 or more

Part 2: Trigonometric Subtopic Questionnaire

The topics below are sometimes included in trigonometry. Check the response code which describe the treatment of each topic in your class.

Response Code A

1. taught for the first time
2. reviewed and then extended
3. reviewed only
4. assumed as prerequisite knowledge
5. not taught and not assumed as prerequisite knowledge

For each topic Taught or Reviewed (ie. if you checked response 1, 2, or 3 under Response Code A) select two numbers from Response Code B which show the **TWO** primary reasons for including it in your course.

Response Code

1. well known to me
2. in Math 10-20-30 Curriculum Guide (1983)
3. easy for students to understand
4. enjoyed by students
5. related to math of prior grades
6. useful for further study in math
7. I was taught it was appropriate
8. assumed to be on the Mathematics Diploma Examination
9. emphasized in student text
10. part of the elective component of Math 30

For each topic that you assumed as prerequisite knowledge (ie. if you checked response 4 under Response Code A) neither Response Code A or B applies.

For each topic Neither Taught Nor Reviewed (ie. if you checked 5 under Response Code A) select two numbers from Response Code C which show the **TWO** primary reasons for excluding it from your course.

Response Code C

1. not well known to me
2. not in Math 10-20-30 Curriculum Guide (1983)
3. hard for students to understand
4. disliked by students
5. not related to math of prior grades
6. not useful for further study in math
7. I was taught it was inappropriate
8. not assumed to be on the Mathematics Diploma Examination
9. not emphasized in student text
10. not part of the elective component of Math 30

	<u>Response Code B</u>		<u>Response Code C</u>	
	First Primary Reason	Second Primary Reason	First Primary Reason	Second Primary Reason

Subtopics

1. Review terms associated with a rectangular coordinate system.

A 1 ___ 2 ___ 3 ___ 4 ___ 5 ___ ___ ___ ___ ___

2. Theorem of Pythagoras

A 1 ___ 2 ___ 3 ___ 4 ___ 5 ___ ___ ___ ___ ___

3. Find coordinates of points in the rectangular coordinate system.

A 1__2__3__4__5__ ____ ____ ____

4. The relationship between degrees, minutes and decimal degrees.

A 1__2__3__4__5__ ____ ____ ____

5. Describe circular path using the initial point, terminal point and directed distance of the path.

A 1__2__3__4__5__ ____ ____ ____

6. Define the unit circle
 $x^2 + y^2 = 1$

A 1__2__3__4__5__ ____ ____ ____

7. Determine coordinates of points on the unit circle.

A 1__2__3__4__5__ ____ ____ ____

8. Define trigonometric functions using a wrapping function with the unit circle.

A 1__2__3__4__5__ ____ ____ ____

9. Determine the trigonometric ratios in terms of coordinates of points on the unit circle.

A 1__2__3__4__5__ ____ ____ ____

10. Find the domain and range of the six trigonometric ratios.

A 1__2__3__4__5__ ____ ____ ____

11. Solve simple trigonometric equations (eg. $2 \sin^2 \theta + \sin \theta = 1$)

A 1 ___ 2 ___ 3 ___ 4 ___ 5 ___ ___ ___ ___

12. Given the value of one of the trigonometric ratios, evaluate the other five trigonometric ratios.

A 1 ___ 2 ___ 3 ___ 4 ___ 5 ___ ___ ___ ___

13. Derive and apply the quotient relations $\tan \theta = \sin \theta / \cos \theta$, $\cot \theta = \cos \theta / \sin \theta$.

A 1 ___ 2 ___ 3 ___ 4 ___ 5 ___ ___ ___ ___

14. Derive and apply the reciprocal relations $\csc \theta = 1/\sin \theta$, $\sec \theta = 1/\cos \theta$, $\cot \theta = 1/\tan \theta$.

A 1 ___ 2 ___ 3 ___ 4 ___ 5 ___ ___ ___ ___

15. Derive and apply the Pythagorean relations $\sin^2 \theta + \cos^2 \theta = 1$, $1 + \tan^2 \theta = \sec^2 \theta$, $1 + \cot^2 \theta = \csc^2 \theta$.

A 1 ___ 2 ___ 3 ___ 4 ___ 5 ___ ___ ___ ___

16. Derive and apply the negative arc formulas: $\sin(-\theta) = -\sin \theta$, $\cos(-\theta) = \cos \theta$, $\tan(-\theta) = -\tan \theta$.

A 1 ___ 2 ___ 3 ___ 4 ___ 5 ___ ___ ___ ___

17. Derive and apply the sum formulas:
 $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
 $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

A 1 ___ 2 ___ 3 ___ 4 ___ 5 ___ ___ ___ ___

18. Derive and apply the complimentary arc formulas: $\cos(\pi/2 - \theta) = \sin \theta$
 $\sin(\pi/2 - \theta) = \cos \theta$.

A 1 2 3 4 5 _____

19. Draw and identify graphs of the sine, cosine and tangent functions.

A 1 2 3 4 5 _____

20. Draw and identify graphs of the secant, cosecant, and cotangent functions.

A 1 2 3 4 5 _____

21. Draw general trigonometric functions such as
 $y = A \sin(Bx + C)$

A 1 2 3 4 5 _____

22. Introduce concept of phase shift, amplitude and period.

A 1 2 3 4 5 _____

23. Define periodic function and state the periods of $\sin \theta$, $\cos \theta$ and $\tan \theta$.

A 1 2 3 4 5 _____

24. Define radian measure.

A 1 2 3 4 5 _____

25. Convert degree measure to radian measure and vice versa.

A 1 2 3 4 5 _____

26. Apply radian measure in practical problems, such as angular velocity.

A 1 ___ 2 ___ 3 ___ 4 ___ 5 ___ ___ ___ ___

27. Determine exact values of trigonometric ratios of 0° , 30° , 45° , 60° and 90° .

A 1 ___ 2 ___ 3 ___ 4 ___ 5 ___ ___ ___ ___

28. Determine the value of a trigonometric ratio of any angle (eg. $\sin(140^\circ) = \sin(90^\circ + 50^\circ)$).

A 1 ___ 2 ___ 3 ___ 4 ___ 5 ___ ___ ___ ___

29. Solution of right triangles given sides or angles.

A 1 ___ 2 ___ 3 ___ 4 ___ 5 ___ ___ ___ ___

30. Solve oblique triangles by using the Sine law and/or Cosine law.

A 1 ___ 2 ___ 3 ___ 4 ___ 5 ___ ___ ___ ___

31. Apply the Sine law and Cosine law to practical problems.

A 1 ___ 2 ___ 3 ___ 4 ___ 5 ___ ___ ___ ___

32. Solve problems involving areas of regular polygons.

A 1 ___ 2 ___ 3 ___ 4 ___ 5 ___ ___ ___ ___

Part 3: Trigonometry Objectives Emphasis Survey

The following are the content objectives of the **Trigonometry Unit** of the Mathematics 30 curriculum (see the Alberta Education Program of Studies). A five-point rating scale is provided after each objective. As shown below, "1" signifies "no emphasis" and "5" signifies "heavy emphasis".

no emphasis 1.....2.....3.....4.....5 heavy emphasis

For each item, circle the number which represents the emphasis you place on that content objective in your own Mathematics 30 class.

1. Maintain previously developed skills.
1.....2.....3.....4.....5.....
2. Describe circular paths using the initial point and directed distance of the path.
1.....2.....3.....4.....5.....
3. Define the unit circle $x^2 + y^2 = 1$.
1.....2.....3.....4.....5.....
4. Determine coordinates of points on the unit circle.
1.....2.....3.....4.....5.....
5. Define the trigonometric ratios in terms of coordinates of points on the unit circle.
1.....2.....3.....4.....5.....

6. Find the domain and range of the six trigonometric ratios.

1.....2.....3.....4.....5.....

7. Solve simple trigonometric equations.

1.....2.....3.....4.....5.....

8. Given the value of one of the trigonometric ratios, evaluate the other five trigonometric ratios.

1.....2.....3.....4.....5.....

9. Derive and apply the following identities:

- a. Quotient relations:

$$\tan \theta = \sin \theta / \cos \theta, \cot \theta = \cos \theta / \sin \theta$$

- b. Reciprocal relations:

$$\csc \theta = 1/\sin \theta, \sec \theta = 1/\cos \theta,$$

$$\cot \theta = 1/\tan \theta$$

- c. Pythagorean relations:

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

1.....2.....3.....4.....5.....

- *10. Derive and apply the following identities:

- a. Negative arc formulas:

$$\sin (-\theta) = -\sin \theta$$

$$\cos (-\theta) = \cos \theta$$

$$\tan (-\theta) = -\tan \theta$$

b. Sum formulas:

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

c. Complementary arc formulas:

$$\cos (\pi / 2 - \theta) = \sin \theta$$

$$\sin (\pi / 2 - \theta) = \cos \theta$$

1.....2.....3.....4.....5.....

11. Draw and identify graphs of the sine, cosine and tangent functions.

1.....2.....3.....4.....5.....

12. Define periodic function and state the periods of sine, cosine and tangent.

1.....2.....3.....4.....5.....

13. Define radian measure.

1.....2.....3.....4.....5.....

14. Convert degree measure to radian measure and vice versa.

1.....2.....3.....4.....5.....

15. Determine exact values of trigonometric ratios of 0° , 30° , 45° , 60° and 90° .

1.....2.....3.....4.....5.....

16. Determine the value of a trigonometric ratio of any angle.

1.....2.....3.....4.....5.....

17. Solve oblique triangles by using the sine law and/or cosine law.

1.....2.....3.....4.....5.....

18. Apply the sine law and cosine law to practical problems.

1.....2.....3.....4.....5.....

19. Solve problems involving areas of regular polygons.

1.....2.....3.....4.....5.....

Part 4: Trigonometric Formula Questionnaire

Several formulas (theorems) involving the trigonometric functions can be developed during the study of trigonometry. For each of the following formulas

CHECK the ONE response which best describes the manner in which you **INITIALLY** presented the formula to your class.

Response Code A

1. I gave a formal proof for the formula.
2. I stated the formula and gave an informal justification (graphic or verbal).
3. I stated the formula and used it without any justification.
4. I did not cover the formula or discuss it with my class.

If you gave a formal proof for the formula, (ie. if you checked number 1 under response code A) **CHECK the ONE** response code below which indicates your primary reason.

Response Code B

1. textbook gave proof.
2. proof unifies basic ideas.
3. interesting examples of mathematical reasoning.

CHECK the ONE response code below which best describes your expectation of student mastery for this formula (theorem). (Omit if you did not cover the formula).

Response Code C

1. prove and apply formula.
2. informal justification and apply.
3. recall and apply formula.
4. apply formula when given.

Response Code B Response Code C

1. $\cos (x-y) = \cos x \cdot \cos y + \sin x \cdot \sin y$

A 1__2__3__4__

1__2__3__

1__2__3__4__

2. $\cos (-x) = \cos x$

A 1__2__3__4__

1__2__3__

1__2__3__4__

3. $\cos (\pi /2 - x) = \sin x$

A 1__2__3__4__

1__2__3__

1__2__3__4__

4. $\sin (-x) = -\sin x$

A 1__2__3__4__

1__2__3__

1__2__3__4__

5. $\sin (\pi /2 - x) = \cos x$

A 1__2__3__4__

1__2__3__

1__2__3__4__

6. $\cos (x + y) = \cos x \cdot \cos y - \sin x \cdot \sin y$

A 1__2__3__4__

1__2__3__

1__2__3__4__

7. $\sin (x + y) = \sin x \cdot \cos y + \cos x \cdot \sin y$

A 1__2__3__4__

1__2__3__

1__2__3__4__

$$8. \sin(x - y) = \sin x \cdot \cos y - \cos x \cdot \sin y$$

A 1__2__3__4__

1__2__3__

1__2__3__4__

$$9. \cos 2x = 2 \cos^2 x - 1$$

A 1__2__3__4__

1__2__3__

1__2__3__4__

$$10. \sin 2x = 2 \sin x \cdot \cos x$$

A 1__2__3__4__

1__2__3__

1__2__3__4__

$$11. \tan 2x = (2 \tan x)/(1 - \tan^2 x)$$

A __2__3__4__

1__2__3__

1__2__3__4__

$$12. \cos x/2 = \pm \{(1 + \cos x)/2\}^{1/2}$$

A 1__2__3__4__

1__2__3__

1__2__3__4__

$$13. \tan(x+y) = (\tan x + \tan y)/(1 - \tan x \cdot \tan y)$$

A 1__2__3__4__

1__2__3__

1__2__3__4__

$$14. \tan(x-y) = (\tan x - \tan y)/(1 + \tan x \cdot \tan y)$$

A 1__2__3__4__

1__2__3__

1__2__3__4__

$$15. a/\sin(A) = b/\sin(B) = c/\sin(C)$$

A 1__2__3__4__

1__2__3__

1__2__3__4__

16. $\sin x/2 = \pm \{(1 - \cos x)/2\}^{1/2}$

A 1__2__3__4__

1__2__3__

1__2__3__4__

17. $\tan (1/2 x) = (1 - \cos x)/\sin x$

A 1__2__3__4__

1__2__3__

1__2__3__4__

18. $a^2 = b^2 + c^2 - 2bc \cos (A)$

A 1__2__3__4__

1__2__3__

1__2__3__4__

19. $\tan (-x) = -\tan x$

A 1__2__3__4__

1__2__3__

1__2__3__4__

20. $\tan x = \sin x / \cos x$

A 1__2__3__4__

1__2__3__

1__2__3__4__

21. $\cot x = \cos x / \sin x$

A 1__2__3__4__

1__2__3__

1__2__3__4__

22. $\csc x = 1/\sin x$

A 1__2__3__4__

1__2__3__

1__2__3__4__

23. $\sec x = 1/\cos x$

A 1__2__3__4__

1__2__3__

1__2__3__4__

24. $\cot x = 1/\tan x$

A 1__2__3__4__

1__2__3__

1__2__3__4__

25. $\sin^2 x + \cos^2 x = 1$

A 1__2__3__4__

1__2__3__

1__2__3__4__

26. $1 + \tan^2 x = \sec^2 x$

A 1__2__3__4__

1__2__3__

1__2__3__4__

27. $1 + \cot^2 x = \csc^2 x$

A 1__2__3__4__

1__2__3__

1__2__3__4__

Part 5: Trigonometric Interpretation Questionnaire

Please respond to the four interpretations in accordance with the three response codes provided below:

Response Code A

1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used, but not emphasized
3. Not used

Response Code B

1. I knew this interpretation quite well.
2. The interpretation is emphasized in the Alberta Curriculum Guide.
3. The interpretation is easy for students to understand.
4. The interpretation is enjoyed by the students.
5. This interpretation relates closely to math studied in prior grades.
6. This interpretation is useful in the study of math in the future.
7. I was taught that this was the appropriate interpretation.
8. This interpretation was presented by the textbook.
9. I assumed this interpretation to be on the Mathematics Diploma Examination.

Response Code C

1. I did not know this interpretation well.
2. The interpretation is not emphasized in the Alberta Curriculum Guide.
3. The interpretation is difficult for students to understand.
4. The interpretation is disliked by the students.
5. This interpretation does not relate closely to the math studied in prior grades.
6. This interpretation is not useful in the study of math in the future.
7. I was taught that this was an inappropriate interpretation.
8. This interpretation was not presented by the textbook.
9. I did not assume it to be on the Mathematics Diploma Examination.

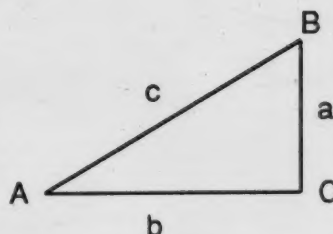
1. RATIO INTERPRETATION:

The three basic trigonometric functions are defined and illustrated as ratios of the lengths of sides of a right triangle.

$$\sin A = a/c \quad \csc A = c/a$$

$$\cos A = b/c \quad \sec A = c/b$$

$$\tan A = a/b \quad \cot A = b/a$$



A 1 2 3

B 1 2 3 4 5 6 7 8 9

C 1 2 3 4 5 6 7 8 9

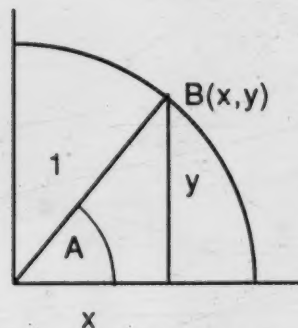
2. UNIT REFERENCE CIRCLE

The three functions are defined and illustrated as in the ratio interpretation except the triangle has hypotenuse 1 and point B is located on a unit radius circle with center at the origin of the coordinate plane. Here, the functions map the real angle measures into real numbers.

$$\sin A = y \quad \csc A = 1/y$$

$$\cos A = x \quad \sec A = 1/x$$

$$\tan A = y/x \quad \cot A = x/y$$



A 1 2 3

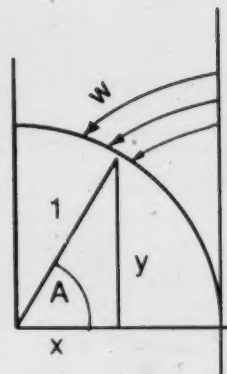
B 1 2 3 4 5 6 7 8 9

C 1 2 3 4 5 6 7 8 9

3. WRAPPING FUNCTION

First, a wrapping function W is defined as mapping the real number line onto the circumference of the unit circle. The trigonometric functions are then defined so that they map the real number angle measures (modulo 2π) to the real numbers.

$$\begin{aligned}\sin A &= y & \csc A &= 1/y \\ \cos A &= x & \sec A &= 1/x \\ \tan A &= y/x & \cot A &= x/y\end{aligned}$$



A 1 2 3

B 1 2 3 4 5 6 7 8 9

C 1 2 3 4 5 6 7 8 9

4. LINE REPRESENTATION OF TRIGONOMETRIC FUNCTIONS

The six trigonometric functions are represented by the length of a line segment representing the numerator of that ratio.

segment $\overline{OD} = \cos A$

$\overline{DR} = \sin A$

$\overline{MC} = \tan A$

$\overline{OC} = \sec A$

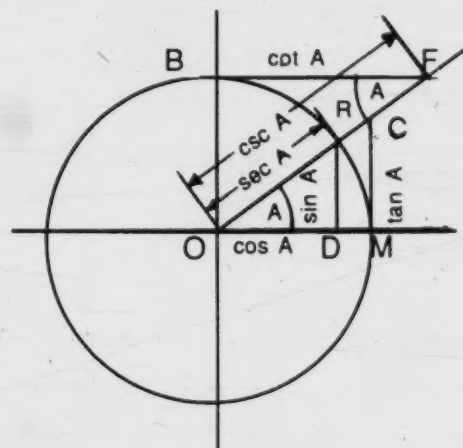
$\overline{OF} = \csc A$

$\overline{BF} = \cot A$

$\overline{OB} = 1$

$\overline{OM} = 1$

$\overline{OR} = 1$



A 1 2 3

B 1 2 3 4 5 6 7 8 9

C 1 2 3 4 5 6 7 8 9

Part 6: Skill Emphasis Survey

Indicate the level of emphasis placed on the following skills by checking the response code below:

Response Codes:

1. Primary importance in my class
2. Secondary importance in my class
3. Little importance in my class
4. No importance in my class
5. Students already had this skill mastered

A. Skill in solving for missing parts of right triangles.

1 _____ 2 _____ 3 _____ 4 _____ 5 _____

B. Skill in solving for missing parts of any triangle.

1 _____ 2 _____ 3 _____ 4 _____ 5 _____

C. Skill in proving trigonometric identities.

1 _____ 2 _____ 3 _____ 4 _____ 5 _____

D. Skill in algebraic techniques for dealing with trigonometry problems.

1 _____ 2 _____ 3 _____ 4 _____ 5 _____

E. Skill in solving equations involving trigonometric functions.

1 _____ 2 _____ 3 _____ 4 _____ 5 _____

F. Skill in graphing trigonometric functions.

1 _____ 2 _____ 3 _____ 4 _____ 5 _____

G. Skill in graphing inverse trigonometric functions.

1 _____ 2 _____ 3 _____ 4 _____ 5 _____

H. Skill in finding the domain and range of trigonometric functions.

1 _____ 2 _____ 3 _____ 4 _____ 5 _____

I. Skill in applying trigonometry to application problems.

1 _____ 2 _____ 3 _____ 4 _____ 5 _____

J. Skill in the use of electronic devices (eg. hand calculators, micro computers)
for solving problems in trigonometry.

1 _____ 2 _____ 3 _____ 4 _____ 5 _____

Part 7: Teaching Strategies Survey

In the following section you are asked to indicate how frequently you use particular teaching methods in the **Trigonometry Unit** of your Mathematics 30 classroom.

In each item, circle the letter of the appropriate response.

1. Lectures:
 - a. never or rarely
 - b. about once or twice during the unit
 - c. about once or twice during the week
 - d. almost every class period
2. Teacher Demonstrations:
 - a. never or rarely
 - b. about once or twice during the unit
 - c. about once or twice during the week
 - d. almost every class period
3. Class discussion (ie. teacher/class dialogue):
 - a. never or rarely
 - b. about once or twice during the unit
 - c. about once or twice during the week
 - d. almost every class period
4. Small group activities or projects:
 - a. never or rarely
 - b. about once or twice during the unit
 - c. about once or twice during the week
 - d. almost every class period
5. Individual projects:
 - a. never or rarely
 - b. about once or twice during the unit
 - c. about once or twice during the week
 - d. almost every class period
6. Library research:
 - a. never or rarely
 - b. about once or twice during the unit
 - c. about once or twice during the week
 - d. almost every class period

7. Math-lab experiments:

- a. never or rarely
- b. about once or twice during the unit
- c. about once or twice during the week
- d. almost every class period

8. Your own worksheets:

- a. never or rarely
- b. about once or twice during the unit
- c. about once or twice during the week
- d. almost every class period

9. Field trips and excursions:

- a. never or rarely
- b. about once or twice during the unit
- c. about once or twice during the week
- d. almost every class period

10. Drill and practice:

- a. never or rarely
- b. about once or twice during the unit
- c. about once or twice during the week
- d. almost every class period

11. Student-directed class discussions:

- a. never or rarely
- b. about once or twice during the unit
- c. about once or twice during the week
- d. almost every class period

12. Resource person (ie. guest speaker):

- a. never or rarely
- b. about once or twice during the unit
- c. about once or twice during the week
- d. almost every class period

13. Films or videotapes:

- a. never or rarely
- b. about once or twice during the unit
- c. about once or twice during the week
- d. almost every class period

14. Class projects:

- a. never or rarely
- b. about once or twice during the unit
- c. about once or twice during the week
- d. almost every class period

15. Newspaper or magazine articles:

- a. never or rarely
- b. about once or twice during the unit
- c. about once or twice during the week
- d. almost every class period

16. Use of computer by yourself:

- a. never or rarely
- b. about once or twice during the unit
- c. about once or twice during the week
- d. almost every class period

17. Use of computer by students:

- a. never or rarely
- b. about once or twice during the unit
- c. about once or twice during the week
- d. almost every class period

18. Individual learning packages:

- a. never or rarely
- b. about once or twice during the unit
- c. about once or twice during the week
- d. almost every class period

19. Discovery Approach:

- a. never or rarely
- b. about once or twice during the unit
- c. about once or twice during the week
- d. almost every class period

20. Tests or quizzes:

- a. never or rarely
- b. about once or twice during the unit
- c. about once or twice during the week
- d. almost every class period

21. Having students put homework on the blackboard:
 - a. never or rarely
 - b. about once or twice during the unit
 - c. about once or twice during the week
 - d. almost every class period
22. Having students hand in homework for evaluation:
 - a. never or rarely
 - b. about once or twice during the unit
 - c. about once or twice during the week
 - d. almost every class period
23. Checking homework by yourself solving all problems on the blackboard:
 - a. never or rarely
 - b. about once or twice during the unit
 - c. about once or twice during the week
 - d. almost every class period
24. Problem solving of "real life" applications as described in the Mathematics 30 curriculum guide:
 - a. never or rarely
 - b. about once or twice during the unit
 - c. about once or twice during the week
 - d. almost every class period
25. Application of trigonometry to other curricula or subjects.
 - a. never or rarely
 - b. about once or twice during the unit
 - c. about once or twice during the week
 - d. almost every class period
26. Topics from other subjects that are applicable in trigonometry.
 - a. never or rarely
 - b. about once or twice during the unit
 - c. about once or twice during the week
 - d. almost every class period

Part 8: Curriculum Material Selection Survey

Selecting and adapting curriculum materials to meet the needs of a particular classroom situation is an integral part of effective teaching. In the following section you are asked to indicate how important you consider certain factors for selecting of and determining how you use classroom materials in your Mathematics 30 classroom. A five-point rating scale is provided after each item. As shown below, "1" signifies "not important" and "5" signifies "extremely important".

not important 1.....2.....3.....4.....5 extremely important

For each item, circle the number which represents the importance you attach to that factor.

1. What students will need to know in order to succeed in their post-secondary education (eg. University, technical schools, nursing, etc.)
1.....2.....3.....4.....5
2. What students will need to know in order to understand the importance of mathematics in their daily life.
1.....2.....3.....4.....5
3. What society needs by way of informed citizen.
1.....2.....3.....4.....5
4. What students will need to know in order to do well on the mathematics diploma examination.
1.....2.....3.....4.....5
5. What students require in order to understand the structure of mathematics.
1.....2.....3.....4.....5

6. What students require in order to understand the historical development of mathematics.

1.....2.....3.....4.....5

7. What students will enjoy and find personally interesting.

1.....2.....3.....4.....5

8. What students find easy to understand.

1.....2.....3.....4.....5

9. The guidelines given in the Mathematics 30 Curriculum Guide.

1.....2.....3.....4.....5

10. The students' ability.

1.....2.....3.....4.....5

11. The time available.

1.....2.....3.....4.....5

12. Approaches that have worked well for me in the past.

1.....2.....3.....4.....5

13. The material is emphasized in the student text.

1.....2.....3.....4.....5

14. The material builds on or extends concepts covered in previous grades (or activities).

1.....2.....3.....4.....5

15. My own interests, background and preferences.

1.....2.....3.....4.....5

Part 9: Resource Material Survey

The selection and use of appropriate resources, whether in print, non-print or electronic courseware materials is very important to facilitate the teaching and learning process. Please indicate how frequently you use the various resources listed below in teaching the **Trigonometry Unit** of the Mathematics 30 program.

1. Used frequently 2. Used occasional 3. Not used at all

1. Math Is/6.

1 _____ 2 _____ 3 _____

2. Foundations of Mathematics for Tomorrow: Senior.

1 _____ 2 _____ 3 _____

3. Holt Mathematics 6.

1 _____ 2 _____ 3 _____

4. Modern Intermediate Algebra.

1 _____ 2 _____ 3 _____

5. Applied Mathematics for Today: Senior

1 _____ 2 _____ 3 _____

6. Curriculum Guide.

1 _____ 2 _____ 3 _____

7. Magazines or papers (NCTM publications, etc.).

1 _____ 2 _____ 3 _____

8. NCTM Yearbook.

1 _____ 2 _____ 3 _____

9. Worksheets, Exercises and Problems prepared by myself.

1 _____ 2 _____ 2 _____

10. Computer software.

1 _____ 2 _____ 3 _____

11. Other (please specify)

1 _____ 2 _____ 3 _____

Part 10: Time Allocation Questionnaire

The allocation of time to various topics or activities is closely related to the instructional emphasis given to these topics. In this section you are asked to indicate how much time is being spent in each of these activities. Your responses should be rounded to the nearest one half hour. Please note that the sum of the hours for questions 2-13 should not exceed the total under question one.

1. How many hours did you spend in total on the study of trigonometry in your Mathematics 30 class?
_____ hours
2. How many hours of your Mathematics 30 trigonometry unit did you spend on reviewing and maintaining previously learned materials (ie. materials learned prior to Mathematics 30)?
_____ hours
3. How many hours of your Mathematics 30 trigonometry unit did you spend on dealing with the unit circle?
_____ hours
4. How many hours of your Mathematics 30 trigonometry unit did you spend on the study of trigonometric identities?
_____ hours
5. How many hours of your Mathematics 30 trigonometry unit did you spend on dealing with the concept of a wrapping function?
_____ hours
6. How many hours of your Mathematics 30 trigonometry unit did you spend on the applications of trigonometric functions?
_____ hours

7. How many hours of your Mathematics 30 trigonometry unit did you spend on the study of graphing of trigonometric functions?
_____ hours
8. How many hours of your Mathematics 30 trigonometry unit did you spend on studying trigonometric functions as ratios of the length of sides of a right triangle?
_____ hours
9. How many hours of your Mathematics 30 trigonometry unit did you spend on solving equations involving trigonometric functions?
_____ hours
10. How many hours of your Mathematics 30 trigonometry unit did you spend on finding the domain and range of the six trigonometric functions (ie. sin, cos, tan, cot, csc, sec)?
_____ hours
11. How many hours of your Mathematics 30 trigonometry unit did you spend on developing the concept of periodicity?
_____ hours
12. How many hours of your Mathematics 30 trigonometry unit did you spend on developing and applying the sine law and/or cosine law?
_____ hours
13. How many hours of your Mathematics 30 trigonometry unit did you spend on developing elective topics?
_____ hours

APPENDIX C

Attitude Toward Mathematics Test

This test has three sections: (1) Mathematics in School, (2) Mathematics as a Process, and (3) Mathematics and the Student.

Indicate how you feel about each statement by checking the appropriate number in accordance with the response code.

Response Code A

1. Very important
2. Important
3. Undecided
4. Not important
5. Not at all important

Response Code B

1. Like a lot
2. Like
3. Undecided
4. Dislike
5. Dislike a lot

Mathematics in School

1. Checking an answer to a problem by going over it.

A 1 2 3 4 5

B 1 2 3 4 5

2. Memorizing trigonometric formulas and theorems.

A 1 2 3 4 5

B 1 2 3 4 5

3. Solving trigonometric application problems.

A 1 2 3 4 5

B 1 2 3 4 5

4. Using trigonometric tables.

A 1 2 3 4 5

B 1 2 3 4 5

5. Solving trigonometric equations.

A 1 2 3 4 5

B 1 2 3 4 5

6. Proving trigonometric theorems.

A 1 2 3 4 5

B 1 2 3 4 5

7. Graphing trigonometric functions.

A 1 2 3 4 5

B 1 2 3 4 5

8. Using electronic devices for solving problems in trigonometry.

A 1 2 3 4 5

B 1 2 3 4 5

9. Using algebraic techniques for dealing with trigonometric problems.

A 1 2 3 4 5

B 1 2 3 4 5

Mathematics as a Process

Indicate what your level of agreement is with each of the following statements by checking the appropriate number in accordance with the response code.

Response Code

1. Strongly agree
2. Agree
3. Undecided
4. Disagree
5. Strongly disagree

1. Trigonometry is a good unit of study for creative people
1 ___ 2 ___ 3 ___ 4 ___ 5 ___
2. There is little room for originality in solving trigonometric problems.
1 ___ 2 ___ 3 ___ 4 ___ 5 ___
3. Trigonometry helps one to think according to strict rules.
1 ___ 2 ___ 3 ___ 4 ___ 5 ___
4. Estimating is an important skill in trigonometry.
1 ___ 2 ___ 3 ___ 4 ___ 5 ___
5. There are many different ways to solve most trigonometric problems.
1 ___ 2 ___ 3 ___ 4 ___ 5 ___
6. Learning trigonometry involves mostly memorizing.
1 ___ 2 ___ 3 ___ 4 ___ 5 ___
7. In trigonometry, problems can be solved without using rules.
1 ___ 2 ___ 3 ___ 4 ___ 5 ___
8. Trial and error can often be used to solve a trigonometric problem.
1 ___ 2 ___ 3 ___ 4 ___ 5 ___
9. There is always a rule to follow in solving trigonometric problems.
1 ___ 2 ___ 3 ___ 4 ___ 5 ___
10. Trigonometry is a set of rules.
1 ___ 2 ___ 3 ___ 4 ___ 5 ___

11. A trigonometric problem can always be solved in different ways.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____

12. Trigonometry helps one to think logically.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____

Mathematics and the Student

Indicate your personal feelings toward each of the following statements by checking the appropriate number in accordance with the response code below:

Response Code

1. Strongly positive
2. Positive
3. Undecided
4. Negative
5. Strongly negative

1. I really want to do well in trigonometry.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____

2. I am looking forward to taking more mathematics.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____

3. I feel good when I solve a mathematics problem by myself.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____

4. I usually understand what we are talking about in trigonometry.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____

5. I am not so good at trigonometry.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____

6. If I had my choice I would not learn any more mathematics.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____

7. I feel challenged when I am given a difficult mathematics problem.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____

8. I refuse to spend a lot of my own time doing mathematics.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____

9. Mathematics is harder for me than for most people.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____

10. I could never be a good mathematician.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____

11. No matter how hard I try I still do not do well in mathematics.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____

12. I will work a long time in order to understand a new idea in mathematics.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____

13. Working with numbers makes me happy.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____

14. It scares me to have to take mathematics.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____

15. I usually feel calm when doing mathematics problems.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____

16. I think mathematics is fun.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____

17. When I cannot figure out a problem, I feel as though I am lost in a maze and
cannot find my way out.

1 ____ 2 ____ 3 ____ 4 ____ 5 ____